Session 3

Reachability and Controllability. Observability and Reconstructability. Controller and Observer Forms.

Reading Assignment

Rugh, Ch 9, 13, 14 (only Theorem 14.9) (for continuous-time systems) and Ch 25 (for discrete-time systems).

Exercise 3.1 = Rugh 9.1.

Exercise 3.2 = Rugh 9.2

Exercise 3.3 = Rugh 9.4

Exercise 3.4 = Rugh 9.5

Exercise 3.5 = Rugh 9.7

Exercise 3.6

a. Show that $\{A, B\}$ is controllable if and only if $\{PAP^{-1}, PB\}$ is controllable for *some* invertible P.

b. Prove that $\{A, B\}$ is controllable if and only if $\{A - BL, B\}$ is controllable for *some* L.

c. Prove that $\{A, B\}$ is controllable if and only if $\{A, BB^T\}$ is controllable.

Exercise 3.7 There is an alternative to the controller form, the controllability form, resulting in a transformed system of the form (for single input)

$$A_{co} = \begin{bmatrix} & & & \star \\ 1 & & & \star \\ & \ddots & & \vdots \\ & & 1 & \star \end{bmatrix}, \quad b_{co} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Find the transformation P taking the (single input) system to this form. (Hint: What is $\mathcal{C}(A_{co}, b_{co})$?) Also draw figures of the controller and controllability forms.

Exercise 3.8 Consider the discrete time system

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

Determine the reachable subspace and the unobservable subspace. Determine Kalman's decomposition

Exercise 3.9 = Rugh 14.8

Exercise 3.10 = Rugh 25.10

Hand in problems - to be handed in at the exercise session

Handin 3.1: = Rugh 9.9

Handin 3.2: Show that the controllability indices of the following system are $\rho_1 = 3, \rho_2 = 2, \rho_3 = 1$ and use Matlab to transform (A, B) to controller form

$$A = \begin{bmatrix} -11 & 13 & -7 & -7 & -15 & 21 \\ 0 & -1 & 6 & 4 & 0 & -1 \\ -3 & 1 & 3 & 2 & -4 & 2 \\ 6 & -1 & -3 & -2 & 8 & -2 \\ 8 & -10 & 5 & 5 & 11 & -16 \\ 0 & 1 & -3 & -2 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 8 & -1 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \\ -2 & -6 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Handin 3.3: = Rugh 25.3