Session 5

LTV stability. Quadratic Lyapunov functions.

Reading Assignment

Rugh Ch 6,7,12 (skip proofs of 7.8, 12.6 and 12.7),14 (pp240-247), and (22,23,24,28)

Exercise 5.1 = Rugh 6.3 iii+iv

Exercise 5.2 = Rugh 6.11

Exercise 5.3 = Rugh 7.3

Exercise 5.4 = Rugh 8.3

Exercise 5.5 = Rugh 7.6

Exercise 5.6 = Rugh 7.11

Exercise 5.7 = Rugh 7.20

Exercise 5.8 = Rugh 23.2

Hand in problems

Exercise 5.9 = Rugh 8.12 with $F(t) = 0$

Exercise 5.10 Consider the time-varying linear system

$$\dot{x}(t) = \begin{bmatrix} e^{-2t} & e^{-t} - e^{-2t} \\ 0 & e^{-t} \end{bmatrix} x(t) \tag{1}$$

Is the equilibrium $x = 0$ of the system (1) uniformly stable? Is it uniformly exponentially stable?

Hint: Apart from transition matrix computation, you may also try Lyapunov transformation to get a transformed system with a diagonal (time-varying) matrix to determine the stability.

Exercise 5.11 Use e.g. CVX to find a constant Lyapunov matrix Q verifying exponential stability for the system

$$\dot{x}(t) = A(t)x(t)$$

where for each t either $A(t) = A_1$ or $A(t) = A_2$ (i.e. $A(t)$ can jump between A_1 and A_2 at arbitrary times), where

$$A_1 = \begin{bmatrix} -1 & 3 \\ 0 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 3 \\ -1 & -6 \end{bmatrix}$$

What is the best exponential convergence rate $\lambda > 0$ you can guarantee?