LUNDS UNIVERSITET

Welcome to

FRTN10 Multivariable Control

Anton Cervin

Department of Automatic Control

- Founded 1965 by Karl Johan Åström (IEEE Medal of Honor)
- Approx. 45 employees
- Education for B, BME, C, D, E, F, I, K, M, N, Pi, W
- Research in autonomous systems, distributed control, AI/ML, robotics, cloud control, med tech, automotive systems, . . .

L1: Introduction

1. Course program
2. Course introduction
3. Signals and systems

Administration

Anton Cervin
Course responsible and lecturer

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M:5142

Mika Nishimura
Course administrator

mika@control.lth.se
046-222 87 85
M:5141

Prerequisites

FRT010 Automatic Control, Basie Course or FRTN25 Automatic Process Control is required prior knowledge.

It is assumed that you have taken the basic courses in mathematics, including linear algebra and calculus in several variables, and preferably also a course in systems & transforms.

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Review

First part of Exercise 1 reviews some important control basics:

- Linear system representations
- Bode diagrams
- Block diagrams
- Stability

Course material

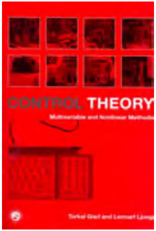
All course material is available in English. Lecture slides, lecture notes, exercise problems, and laboratory assignments are provided on the course homepage

`http://www.control.lth.se/course/FRTN10`

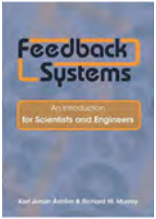
- Lecture slides (also printed and handed out)
- Lecture notes
- Exercise problems with solutions
- Laboratory assignments
- *Automatic Control—Collection of Formulae*
- Old exams

Reading material

Optional reading is provided in Glad & Ljung: *Reglerteori – Flervariabla och olinjära metoder* (Studentlitteratur, 2003) / *Control Theory – Multivariable and Nonlinear Methods* (Taylor & Francis / CRC, 2000, also available as e-book through Lund University Libraries)



A very good online resource for reviewing many of the basic concepts of automatic control is *Feedback Systems: An Introduction to Scientists and Engineers* by Karl Johan Åström and Richard Murray



Lectures

The lectures (30 hours in total) are given by Anton Cervin on Mondays, Tuesdays, and Thursdays.

See the LTH schedule generator for details.

Exercise sessions

The exercise sessions (28 hours in total) are given on Wednesdays and Fridays in two groups (free choice):

Group	*Times*	*Teaching assistant*
1	Wed 13–15, Fri 10–12	Marcus Thelander Andrén
2	Wed 10–12, Fri 13–15	Olle Kjellqvist

Marcus Thelander Andrén

Olle Kjellqvist

Exercises 4, 7, 10, 12 and 13 are computer exercises, held in Lab A in the course lab of Automatic Control LTH, located on the ground floor in the southern part of the M-building. You can also do most of the computer exercises on your own computer if you have Matlab with Control System

Laboratory experiments

The three laboratory sessions (12 hours in total) are mandatory. Booking is done in SAM. You must sign up before the first session starts. Before each session there are pre-lab assignments that must be completed. No reports are required afterwards.

Lab	*Weeks*	*Booking*	*Room*	*Responsible*	*Process*
1	38–39	Sep 5	Lab C	Marcus Thelander Andrén	Flexible linear servo
2	40–41	Sep 16	Lab C	Olle Kjellqvist	Quadruple tank
3	42–43	Sep 30	Lab B	Nils Vreman	MinSeg




Exam

The exam is given on Tuesday October 29 at 08:00–13:00 in Vic3:B-C.

Retake exams are offered in January (unofficial) and April (official).

Lecture slides (with markings/small notes), lecture notes and the Glad & Ljung book are allowed on the exam. You may also bring Automatic Control—Collection of Formulae, standard mathematical tables (e.g., TEFYMA), and a pocket calculator.

Matlab

Matlab is used in the laboratory sessions as well as in the five computer exercise sessions

- Control System Toolbox
- Simulink
- CVX (`http://cvxr.com/cvx`), used in exercise session 12

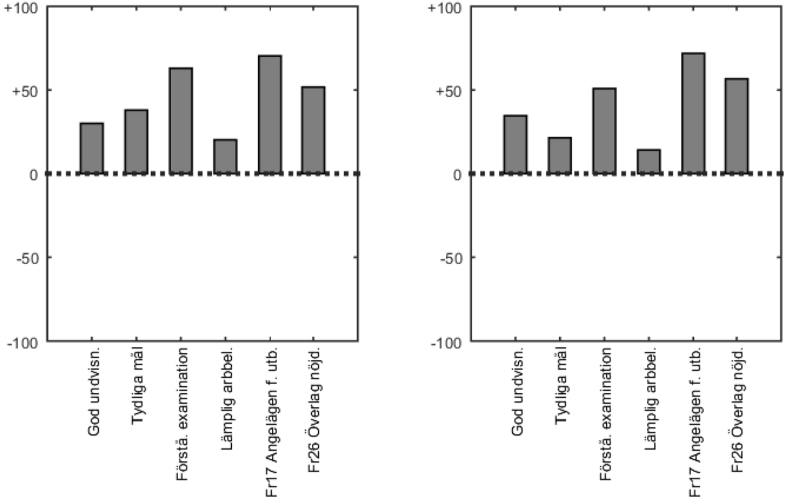
Feedback and Q&A

Feedback mechanisms for improving the course:

- CEQ (reporting / longer time scale)
- Student representatives (short time scale)
  - Election of student representatives ("kursombud")
- Mid-course evaluation

We will use Piazza for Q&A. Take the opportunity to give and take feedback!

CEQ 2017, 2018



Course registration

- Please do the course registration in Ladok as soon as possible!
- If you have not signed up for the course in advance, you need to contact your program planner for late sign-up
- If you have taken the course before and need to be re-registered, send an email to mika@control.lth.se

L1: Introduction

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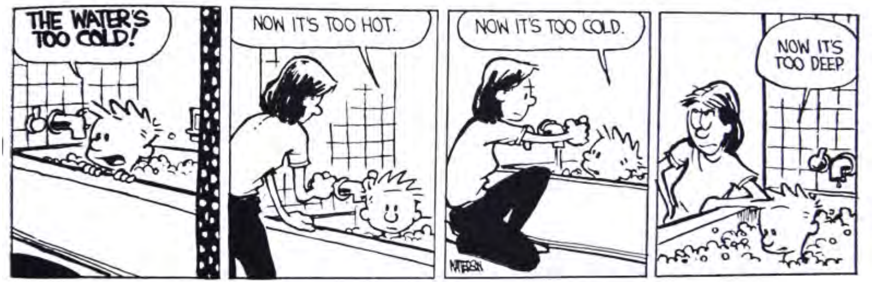
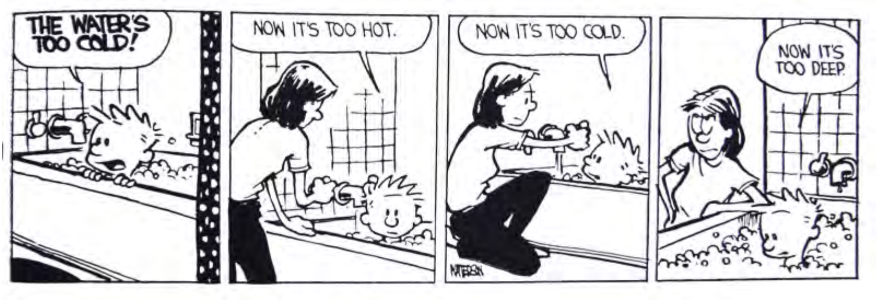
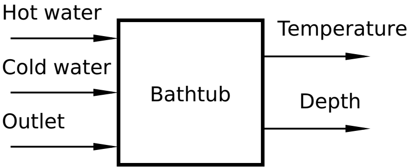
Multivariable control – Example 1

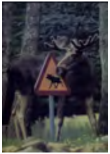


Multivariable control – Example 1

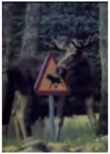


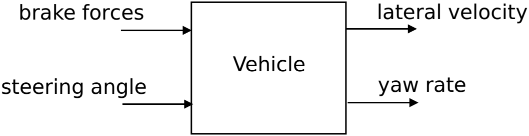


Example 2: Rollover control



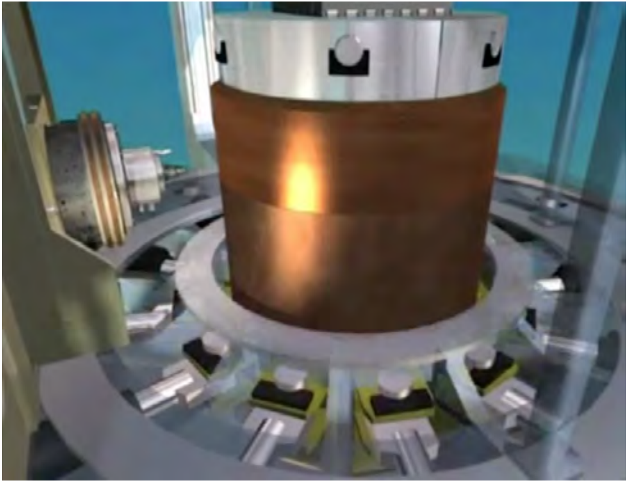
Example 2: Rollover control





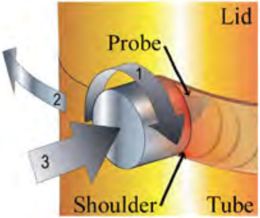
Example 3: Control of friction stir welding

Prototype FSW machine at the Swedish Nuclear Fuel and Waste Management Company (SKB) in Oskarshamn



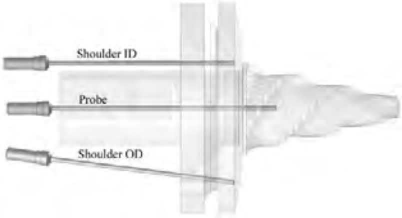
Control of friction stir welding

Control variables:



- Tool rotation speed
- Weld speed
- Axial force

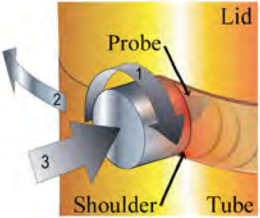
Measurement variables:



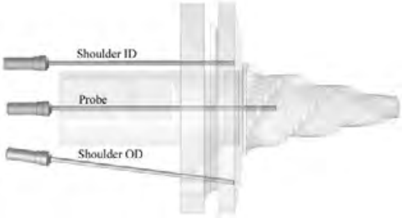
- Temperatures (3 sensors)
- Motor torque
- Shoulder depth

Control of friction stir welding

Control variables:



Measurement variables:



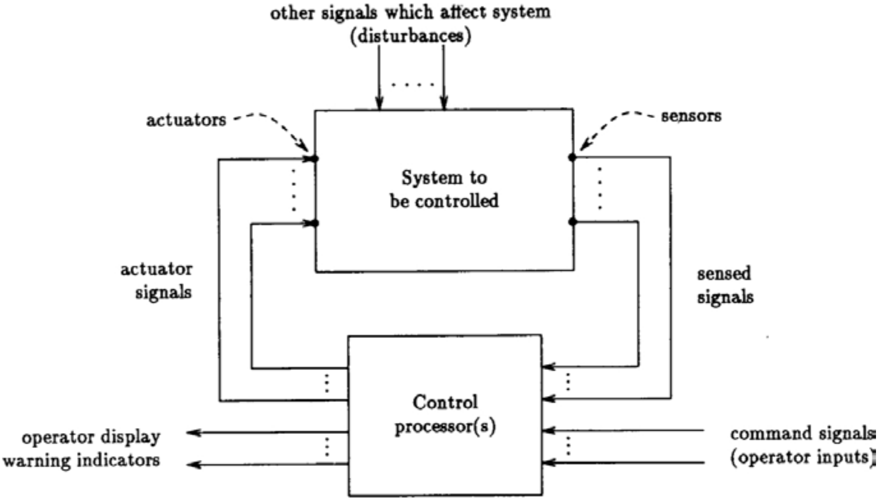
- Tool rotation speed
- Weld speed
- Axial force

- Temperatures (3 sensors)
- Motor torque
- Shoulder depth

Control objectives:

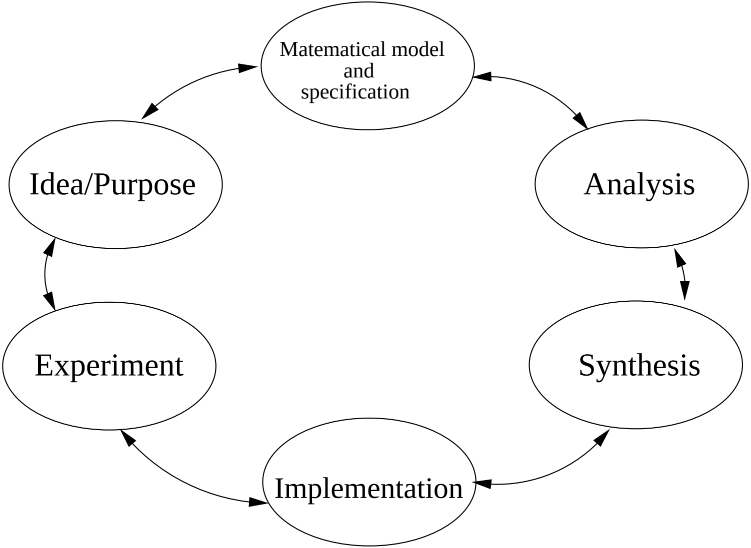
- Keep weld temperature at 845 °C
- Keep shoulder depth at 1 mm

A general control system



[Boyd *et al.*: “Linear Controller Design: Limits of Performance via Convex Optimization”, *Proceedings of the IEEE*, 78:3, 1990]

The control design process



Contents of the course

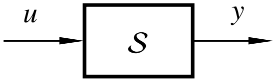
Despite its name, this course is **not only about multivariable control**. You will also learn about:

- sensitivity and robustness
- design trade-offs and fundamental limitations
- stochastic control
- optimization of controllers

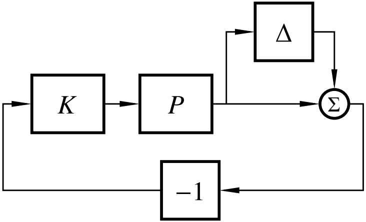
Outline of lectures

L1–L5	Specifications, models and loop-shaping by hand
L6–L8	Limitations on achievable performance
L9–L11	Controller optimization: analytic approach
L12–L14	Controller optimization: numerical approach
L15	Course review

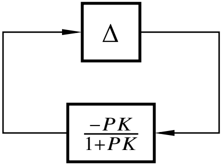
L1: Introduction



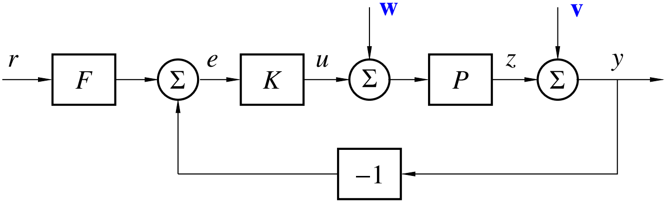
L2: Stability and robustness

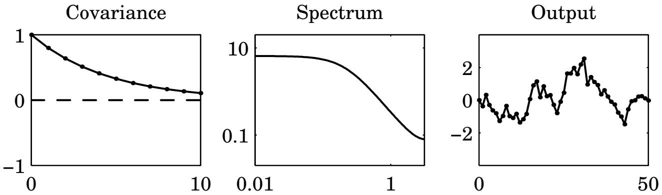


$$\Leftrightarrow$$

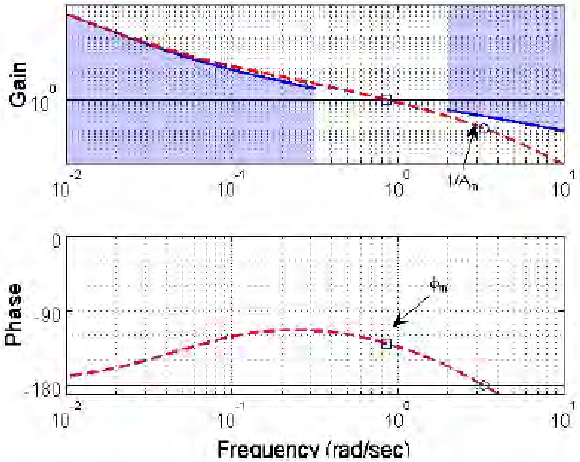


L3: Specifications and disturbance models

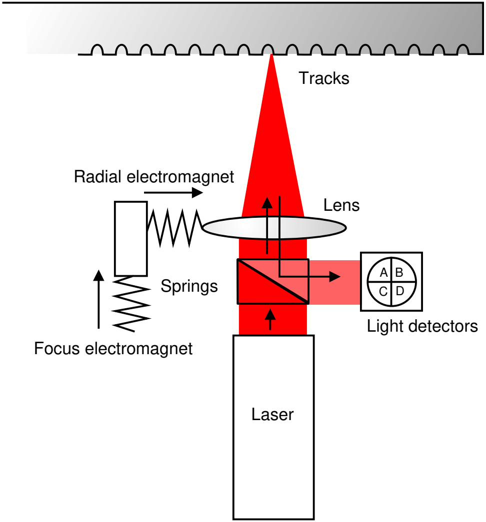


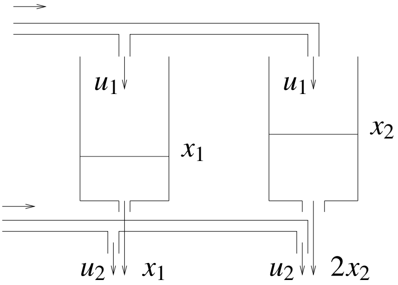


L4: Control synthesis in frequency domain



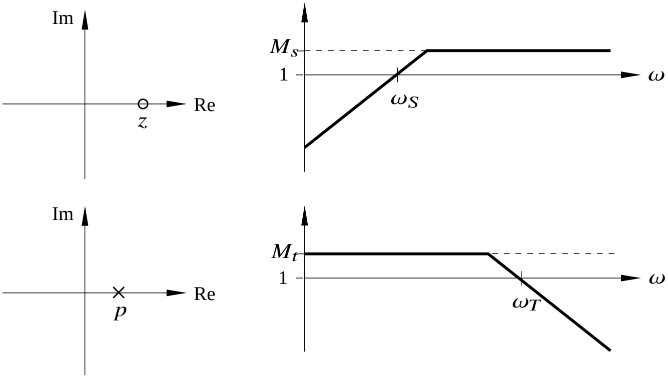
L5: Case study: DVD player



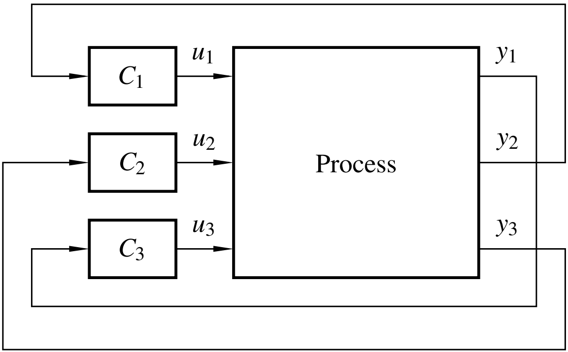


$$G(s) = \begin{pmatrix} \frac{1}{s+2} & 1 \\ \frac{2}{s+2} & 1 \end{pmatrix}$$

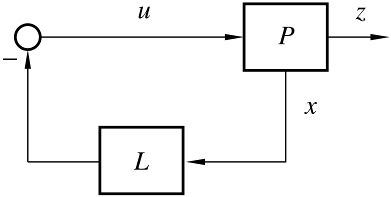
L7: Fundamental limitations



L8: Decentralized control

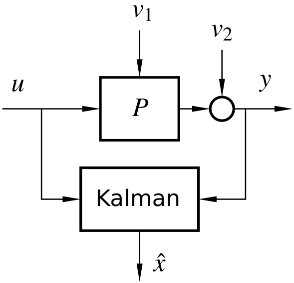


L9: Linear-quadratic control

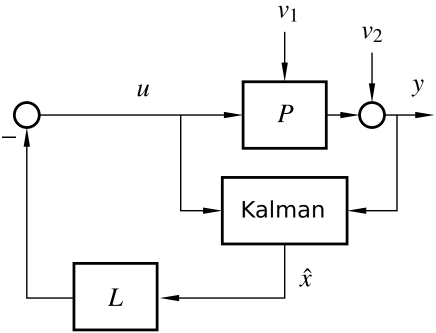


$$\min_L \int_0^\infty \left(x^T Q_1 x + u^T Q_2 u\right) dt$$

L10: Kalman filtering

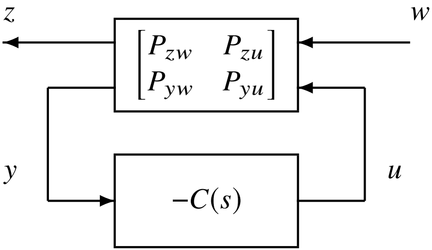


L11: LQG control



$$\min_{K,L} \; \mathop{\mathrm{E}}_{v_1,v_2} \left\{ x^T Q_1 x + u^T Q_2 u \right\}$$

L12: Youla parameterization, internal model control



ALL stabilizing controllers:

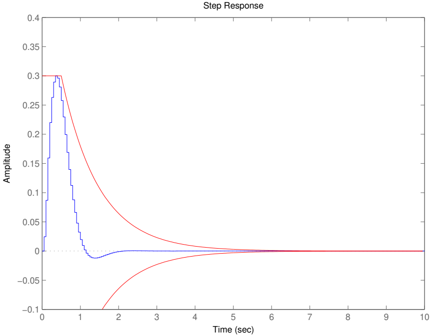
$$C(s) = \left[I - Q(s)P_{yu}(s)\right]^{-1} Q(s)$$

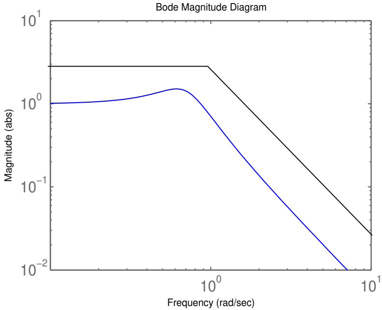
L13: Synthesis by convex optimization

Minimize

$$\int_{-\infty}^{\infty} \left|P_{zw}(i\omega) + P_{zu}(i\omega) \overbrace{\sum_k Q_k \phi_k(i\omega)}^{Q(i\omega)} P_{yw}(i\omega)\right|^2 d\omega$$

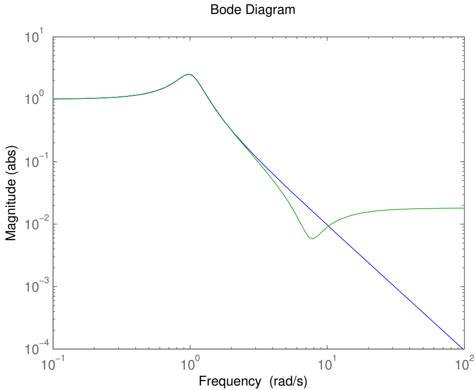
subject to constraints





L14: Controller simplification

$$C(s) = \frac{(s/1.3+1)(s/45+1)}{(s/1.2+1)(s^2+0.4s+1.04)(s/50+1)} \approx \frac{s^2-2.3s+57}{s^2+0.41s+1.1}$$



L1: Introduction

1. Course program
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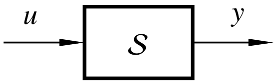
3. Signals and systems
   - System representations
   - Signal norm and system gain

Systems



A **system** is a mapping from the input signal $u(t)$ to the output signal $y(t)$, $-\infty < t < \infty$:

$$y = \mathcal{S}(u)$$

System properties

A system $\mathcal{S}$ is

- **causal** if $y(t_1)$ only depends on $u(t)$, $-\infty < t \leq t_1$,
  non-causal otherwise

- **static** if $y(t_1)$ only depends on $u(t_1)$,
  dynamic otherwise

- **discrete-time** if $u(t)$ and $y(t)$ are only defined for a countable set of discrete time instances $t = t_k,\ k = 0, \pm 1, \pm 2, \ldots$,
  continuous-time otherwise

System properties (cont'd)

A system $\mathcal{S}$ is

- **single-variable** or **scalar** if $u(t)$ and $y(t)$ are scalar signals, **multivariable** otherwise
- **time-invariant** if $y(t) = \mathcal{S}(u(t))$ implies $y(t+\tau) = \mathcal{S}(u(t+\tau))$, **time-varying** otherwise
- **linear** if $\mathcal{S}(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1 \mathcal{S}(u_1) + \alpha_2 \mathcal{S}(u_2)$, **nonlinear** otherwise

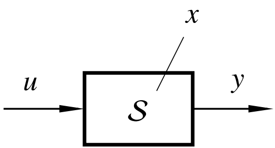
LTI system representations

We will mainly deal with continuous-time **linear time-invariant** (LTI) systems in this course

For LTI systems, the same input–output mapping $\mathcal{S}$ can be represented in a number of equivalent ways:

- linear ordinary differential equation
- linear state-space model
- transfer function
- impulse response
- step response
- frequency response
- ...

State-space models



Linear state-space model:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

Solution:

$$y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$$

Mini-problem 1

$$\dot{x}_1 = -x_1 + 2x_2 + u_1 + u_2 - u_3$$
$$\dot{x}_2 = -5x_2 + 3u_2 + u_3$$
$$y_1 = x_1 + x_2 + u_3$$
$$y_2 = 4x_2 + 7u_1$$

How many states, inputs and outputs?

Determine the matrices A, B, C, D to write the system as

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

Mini-problem 1

Change of coordinates

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

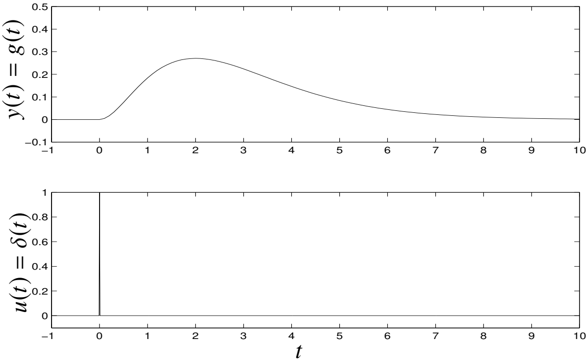
Change of coordinates

$$z = Tx, \quad T \text{ invertible}$$

$$\begin{cases} \dot{z} = T\dot{x} = T(Ax + Bu) & = T(AT^{-1}z + Bu) = TAT^{-1}z + TBu \\ y = Cx + Du & = CT^{-1}z + Du \end{cases}$$

There are infinitely many different state-space representations of the same input–output mapping $y = \mathcal{S}(u)$

Impulse response

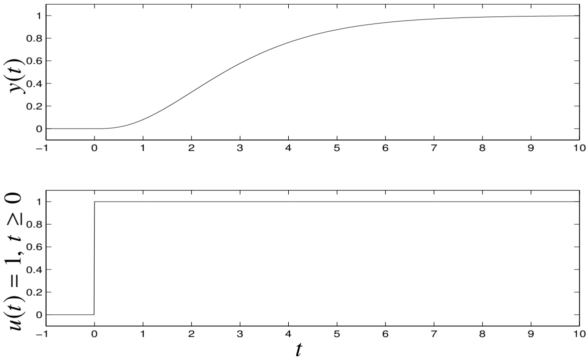


Common experiment in medicine and biology

$$g(t) = \int_0^t Ce^{A(t-\tau)}B\delta(\tau)d\tau + D\delta(t) = Ce^{At}B + D\delta(t)$$

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = (g * u)(t)$$

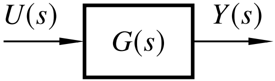
Step response



Common experiment in process industry

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = \int_0^t g(\tau)d\tau$$

Transfer function



$$G(s) = \mathcal{L}\{g(t)\}$$

$$y(t) = (g * u)(t) \quad \Leftrightarrow \quad Y(s) = G(s)U(s)$$

Conversion from state-space form to transfer function:

$$G(s) = C(sI - A)^{-1}B + D$$

Transfer function

A transfer function is **rational** if it can be written as

$$G(s) = \frac{B(s)}{A(s)}$$

where $B(s)$ and $A(s)$ are polynomials in s

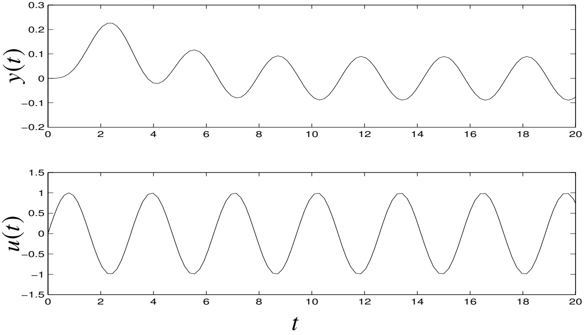
- Example of non-rational function: $G(s) = e^{-sL}$ (time delay)

It is **proper** if $\deg B \leq \deg A$ and **strictly proper** if $\deg B < \deg A$

- Example of non-proper function: $G(s) = s$ (pure derivative)

Note: Only rational and proper transfer functions can be converted to standard state-space form (see Collection of Formulae)

Frequency response



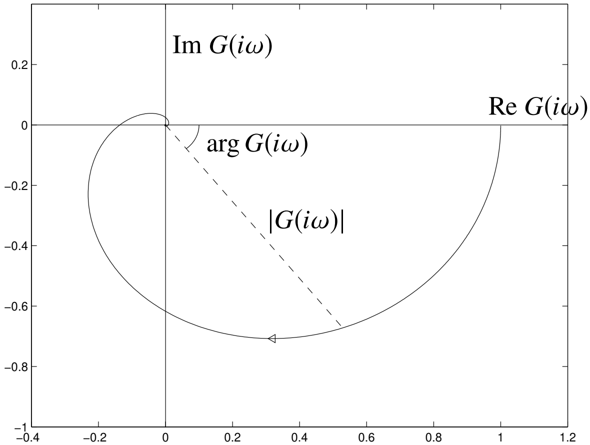
Assume stable transfer function $G = \mathcal{L}g$. Input $u(t) = \sin\omega t$ gives

$$y(t) = \int_0^t g(\tau)u(t-\tau)d\tau = \text{Im}\left[\int_0^t g(\tau)e^{-i\omega\tau}d\tau \cdot e^{i\omega t}\right]$$

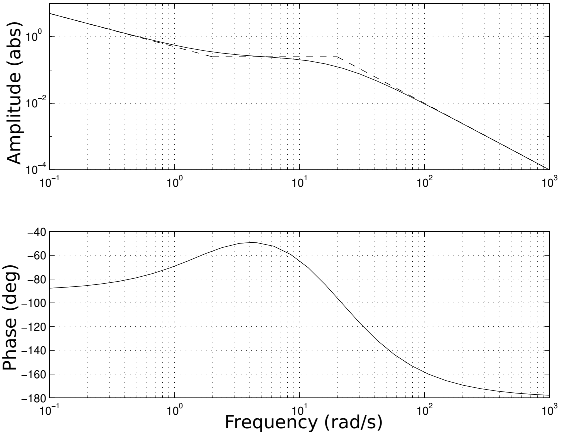
$$[t \longrightarrow \infty] = \text{Im}\left(G(i\omega)e^{i\omega t}\right) = |G(i\omega)|\sin\left(\omega t + \arg G(i\omega)\right)$$

After a transient, also the output becomes sinusoidal

The Nyquist diagram



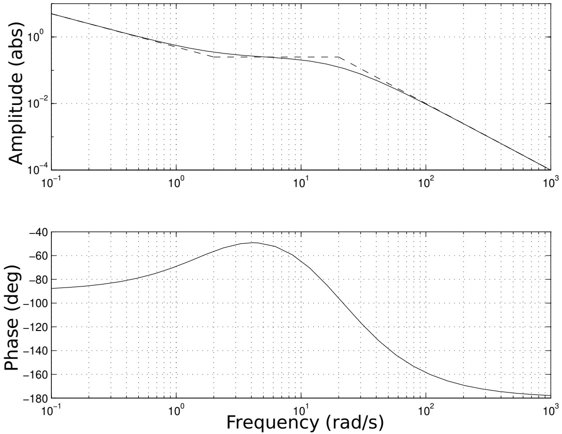
The Bode diagram



$$G = G_1 G_2 G_3 \qquad \begin{cases} \log|G| = \log|G_1| + \log|G_2| + \log|G_3| \\ \arg G = \arg G_1 + \arg G_2 + \arg G_3 \end{cases}$$

Each new factor enters additively

The Bode diagram

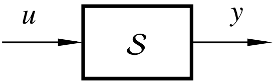


$$G = G_1 G_2 G_3 \qquad \begin{cases} \log|G| = \log|G_1| + \log|G_2| + \log|G_3| \\ \arg G = \arg G_1 + \arg G_2 + \arg G_3 \end{cases}$$

Each new factor enters additively

Hint: Set Matlab units
`>> ctrlpref`

Signal norm and system gain



How to quantify

- the "size" of the signals u and y
- the "maximum amplification" between u and y

Signal norm

The L_2 norm of a signal $y(t) \in \mathbb{R}^n$ is defined as

$$\|y\| = \sqrt{\int_0^\infty |y(t)|^2 dt}$$

By Parseval's theorem it can also be expressed as

$$\|y\| = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} |Y(i\omega)|^2 d\omega}$$

System gain

The L_2 (or "L_2-induced") gain of a general system $\mathcal{S}$ with input u and output $\mathcal{S}(u)$ is defined as

$$\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|}{\|u\|}$$

L_2 gain of LTI systems

THEOREM 1.1

Consider a stable LTI system $\mathcal{S}$ with transfer function $G(s)$. Then

$$\|\mathcal{S}\| = \sup_{\omega} |G(i\omega)| := \|G\|_\infty$$

Proof. Let $y = \mathcal{S}(u)$. Then

$$\|y\|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(i\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 |U(i\omega)|^2 d\omega \leq \|G\|_\infty^2 \|u\|^2$$

The inequality is arbitrarily tight when $u(t)$ is a sinusoid near the maximizing frequency.

(How to interpret $|G(i\omega)|$ for matrix transfer functions will be explained in Lecture 2.)

Mini-problem 2

What are the L_2 gains of the following scalar LTI systems?

1. $y(t) = -u(t)$ (a sign shift)
2. $y(t) = u(t-T)$ (a time delay)
3. $y(t) = \int_0^t u(\tau)d\tau$ (an integrator)
4. $y(t) = \int_0^t e^{-(t-\tau)}u(\tau)d\tau$ (a first-order filter)

Mini-problem 2

Summary of Lecture 1

- Course overview
- Review of LTI system descriptions (see also Exercise 1)
- L_2 norm of signals
  - Definition: $\|y\| := \sqrt{\int_0^\infty |y(t)|^2 dt}$
- L_2 gain of systems
  - Definition: $\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|}{\|u\|}$
  - Special case—stable LTI systems: $\|\mathcal{S}\| = \sup_\omega |G(i\omega)| := \|G\|_\infty$ (also known as the "H_∞ norm" of the system)