Predictive Control Exercise 5
Model Predictive Control (MPC)

1. The receding horizon principal involves finding an open-loop control sequence that minimizes a certain cost function over a finite horizon. This procedure is performed each time new measurements are available. The first step in the computed control sequence is used as the control signal. Although the computed control sequences are open-loop sequences, the calculation of a new sequence at each sample can be thought of as providing feedback.

The finite horizon over which the cost function is evaluated is known as the *prediction horizon* p (since the cost function depends on the predicted values of the output given the initial state and future control changes).

The control changes Δu_i are the optimization variables; these are chosen to minimize the cost function. Since the size of the optimization problem (and consequently the difficulty of finding the solution) depend on the number of optimization variables, the *control horizon* m, representing the number of control changes, can be chosen to be less than the prediction horizon p.

2. The constraint must be fulfilled at all time instances k, so we have:

$$-1 \leq y_{k+1} \leq 2$$

but:

$$\hat{y}_{k+1|k} = 3\hat{x}_{k+1|k} = 6\hat{x}_{k|k-1} + 3u_k$$

Rewriting u_k as $u_{k-1} + \Delta u_k$ gives:

$$\hat{y}_{k+1|k} = 6\hat{x}_{k|k-1} + 3u_{k-1} + 3\Delta u_k$$

Substituting this into the constaint gives:

$$-1 \leq 6\hat{x}_{k|k-1} + 3u_{k-1} + 3\Delta u_k \leq 2$$

Using the fact that $\hat{x}_{k|k-1} = 3$ and $u_{k-1} = -1$ gives:

$$-\frac{16}{3} \leq \Delta u_k \leq -\frac{13}{3}$$

as required.

3.

a.

$$S_x = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^p \end{bmatrix}, \quad S_{u-1} \begin{bmatrix} CB \\ CAB + CB \\ \vdots \\ \sum_{j=0}^{p-1} CA^j B \end{bmatrix}$$

$$S_u = \begin{bmatrix} CB & 0 & \dots & 0 \\ CAB + CB & CB & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{j=0}^{p-1} CA^j B & \sum_{j=0}^{p-2} CA^j B & \dots & CB \end{bmatrix}$$

where p is the prediction horizon.

b. From above, we see that S_x has the structure of an observability matrix. $S_x x_k$ represents the component of the predicted future outputs which are obtained from the current state x_k.

S_{u-1} represents the propagation of the previous sample's control signal u_{k-1} through the prediction horizon. Note that if the control changes over the horizon are zero, then the predicted outputs are given by $\mathcal{Y} = S_x x_k + S_{u-1} u_{k-1}$.

S_u is a lower triangular Toeplitz matrix which describes the effects of the sequence of control changes $\Delta\mathcal{U}$ on the predicted outputs.

4.

a. Since the disturbance is constant we have $d_{k+1} = d_k$. Extending the state vector to:

$$x_k^e \begin{bmatrix} x_k \\ d_k \end{bmatrix}$$

allows an augmented model to be constructed:

$$\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ d_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k$$
$$y_k = \begin{bmatrix} C & I \end{bmatrix} \begin{bmatrix} x_k \\ d_k \end{bmatrix}$$

b. The state vector has been extended, and only output measurements are available. Therefore, a state observer is required.

5. Again, the disturbance is constant we have $d_{k+1} = d_k$. Extending the state vector in the same manner as the previous example:

$$x_k^e \begin{bmatrix} x_k \\ d_k \end{bmatrix}$$

allows an augmented model to be constructed:

$$\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ d_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k$$
$$y_k = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x_k \\ d_k \end{bmatrix}$$

Again, a state observer will be needed.