Distributed Localization of Tree-Structured Scattered Sensor Networks

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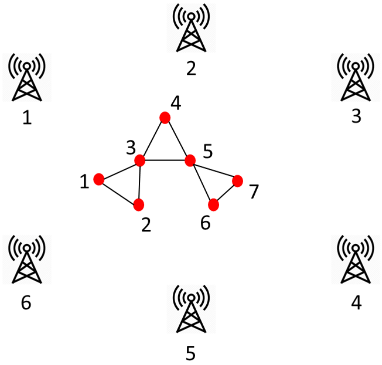
Localization Problem



N sensors of which m anchors; $V_r = \{1, \dots, N\}$

Range measurements described using undirected connected graph $G_r(V_r, \mathcal{E}_r)$

Edge $(i, j) \in \mathcal{E}_r$ if and only if a range measurement between sensors i and j.

Assumptions

Assume $G_r(V_r, \mathcal{E}_r)$ connected with few edges and similarly for the chordal embedding $\bar{G}_r(V_r, \bar{\mathcal{E}}_r)$.

This graph can then be represented using its clique tree. For its cliques $\mathbf{C}_{\bar{G}_r} = \{C_1, \ldots, C_q\}$, we have $|C_i| \ll N$.

We call such sensor networks **tree-structured scattered.**

Measurements

Inter-sensor range measurements:

$$\mathcal{R}_{ij} = \mathcal{D}_{ij} + E_{ij}, \quad j \in \text{Ne}_r(i)$$

$\mathcal{D}_{ij} = \|x_s^i - x_s^j\|_2$ sensor distance; $E_{ij} \sim \mathcal{N}(0, \Sigma_{ij}^r)$ measurement noise

Anchor range measurements:

$$\mathcal{Y}_{ij} = \mathcal{Z}_{ij} + V_{ij}, \quad j \in \text{Ne}_a(i)$$

$\mathcal{Z}_{ij} = \|x_s^i - x_a^j\|_2$ anchor-sensor distance; $V_{ij} \sim \mathcal{N}(0, \Sigma_{ij}^a)$ measurement nose

Maximum Likelihood Problem

$$X^*_{\mathrm{ML}} = \operatorname*{argmin}_X \left\{ \sum_{i=1}^{N} \left(\sum_{j \in \mathrm{Ne}_r(i); i<j} \frac{1}{\Sigma^r_{ij}} \left(\mathcal{D}_{ij}(x^i_s, x^j_s) - \mathcal{R}_{ij} \right)^2 + \sum_{j \in \mathrm{Ne}_a(i)} \frac{1}{\Sigma^a_{ij}} \left(\mathcal{Z}_{ij}(x^i_s, x^j_a) - \mathcal{Y}_{ij} \right)^2 \right) \right\}, \quad (1)$$

where $X = \begin{bmatrix} x^1_s & \dots & x^N_s \end{bmatrix} \in \mathbb{R}^{d \times N}$ with $d = 2$ or $d = 3$

Equivalent Formulation

With

$$f(\Lambda, \Xi, D, Z) := \sum_{i=1}^{N} \left(\sum_{j \in \mathrm{Ne}_r(i); i<j} \frac{1}{\Sigma_{ij}^r} (\Lambda_{ij} - 2D_{ij}\mathcal{R}_{ij} + \mathcal{R}_{ij}^2) + \sum_{j \in \mathrm{Ne}_a(i)} \frac{1}{\Sigma_{ij}^a} (\Xi_{ij} - 2Z_{ij}\mathcal{Y}_{ij} + \mathcal{Y}_{ij}^2) \right). \quad (2)$$

equivalent formulation is

$$\underset{X,S,\Lambda,\Xi,D,Z}{\text{minimize}} \quad f(\Lambda, \Xi, D, Z) \quad (3a)$$

$$\left. \begin{array}{l} S_{ii} + S_{jj} - 2S_{ij} = \Lambda_{ij} \\ \Lambda_{ij} = D_{ij}^2, \quad D_{ij} \geq 0, \; j \in \mathrm{Ne}_r(i), i < j \end{array} \right\}, \; i \in \mathbb{N}_N \quad (3b)$$

$$\left. \begin{array}{l} S_{ii} - 2(x_s^i)^T x_a^j + \|x_a^j\|_2^2 = \Xi_{ij} \\ \Xi_{ij} = Z_{ij}^2, \quad Z_{ij} \geq 0, \; j \in \mathrm{Ne}_a(i) \end{array} \right\}, \; i \in \mathbb{N}_N \quad (3c)$$

$$S = X^T X. \quad (3d)$$

Key Observation

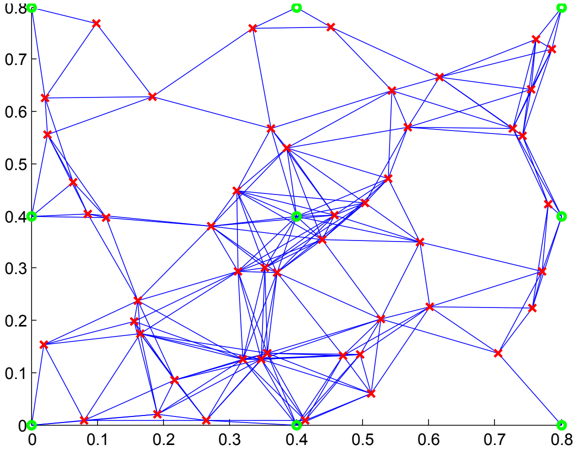
The constraint $S = X^T X$ can equivalently be written

$$S \succeq 0, \quad S_{ij} = (x_s^i)^T x_s^j, \; \forall \, (i,j) \in \mathcal{E}_r \cup \{(i,i) \mid i \in V_r\}$$

Other researchers have performed Semidefinite Programming (SDP) relaxation directly on the problem in the previous slide, used the equivalent constraint description, and then finally matrix completion techniques do develop efficient **centralized** solver.

We instead first apply the equivalent constraint description, then apply matrix completion techniques, and finally SDP relaxations to obtain efficient **distributed** solver.

Artificial Sensor Network

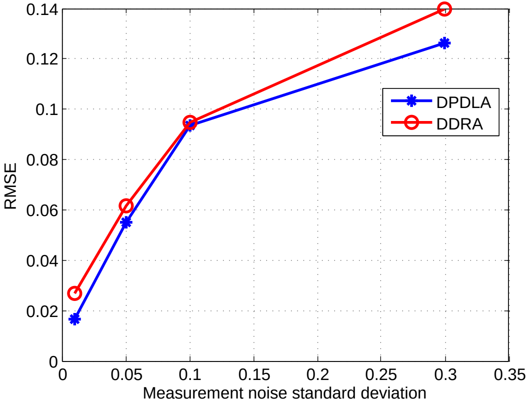


Comparisons for

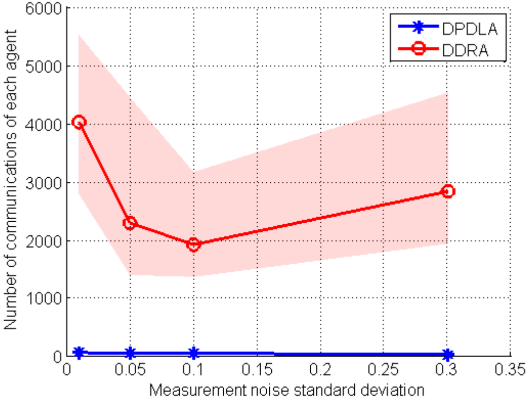
Our algorithm Distributed Primal-Dual Localization Algorithm (DPDLA) and

Distributed Disk Relaxation Algorithm (DDRA) by Soares, Xavier and Gomes, IEEE Trans. Sign. Proc., 2014.

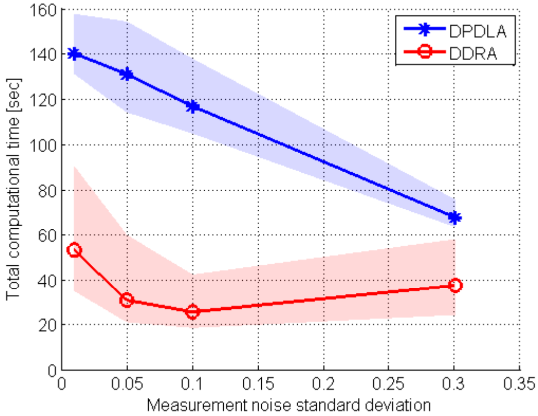
RMSE versus Measurement Noise



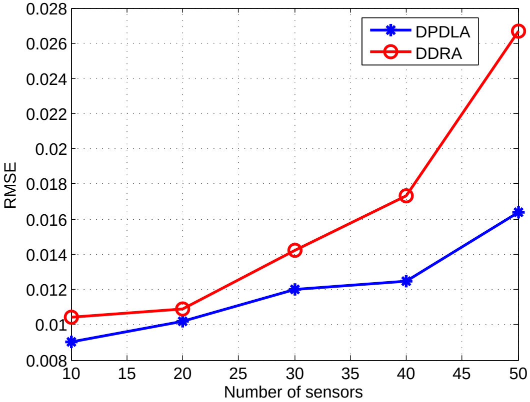
Number of Communications versus Measurement Noise



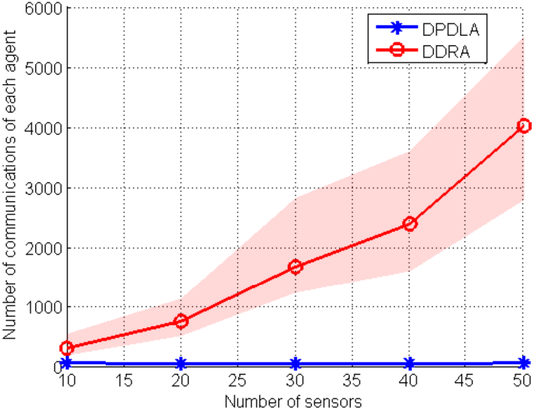
Computational Time versus Measurement Noise



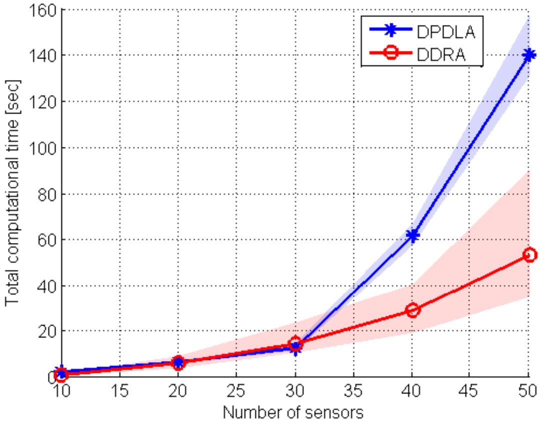
RMSE versus Number of Sensors



Number of Communications versus Number of Sensors



Computational Time versus Number of Sensors



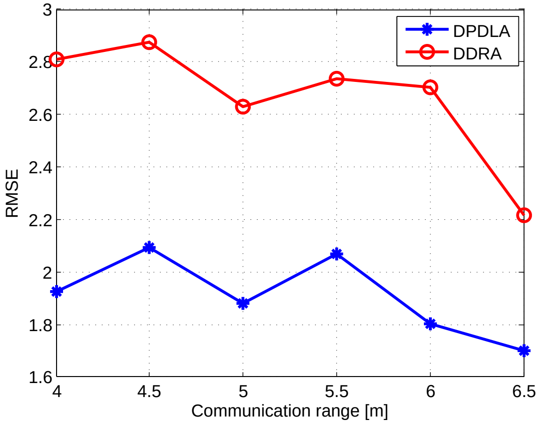
Real Data

Taken from Patwari, Hero III, Perkins, Correal and O'Dea, IEEE Trans. Sign. Proc, 2003

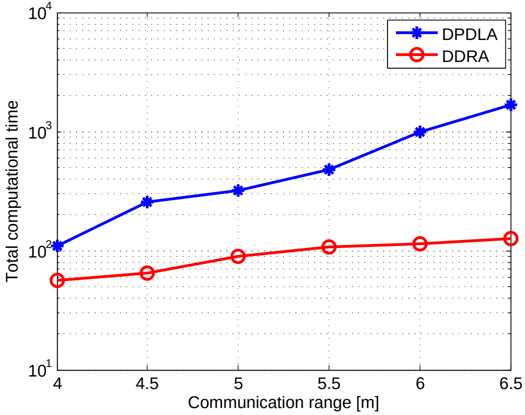
Time of Arrival measurements from 44 sensors of which 4 are anchors from a 13 m by 14 m area.

Biased range measurements with standard deviation 1.82 m.

RMSE versus Communication Range



Total Computational Time versus Communication Range



Summary

- Developed distributed localization algorithm using distributed interior-point methods over trees base on dynamic programming or message passing to compute search directions.
- Needs less communication than other distributed algorithms
- Robust against bias in measurements
- More complicated than first order methods
- Smart clustering of the sensors could potentially increase performance
- Actually all computations could be made at anchors to minimize communication—a design choice!

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Sina Khoshfetrat Pakazad, Emre Özkan, Carsten Fritsche, and Fredrik Gustafsson,

Publications

S. Khoshfetrat Pakazad, E. Özkan, C. Fritsche, A. Hansson, and F. Gustafsson, “Distributed Localization of Tree-Structured Scattered Sensor Networks”, arXiv:1607.04798, 2016

S. Khoshfetrat Pakazad, “Divide and Conquer: Distributed Optimization and Robustness Analysis”, Linköping Studies in Science and Technology, Dissertations, No 1676, 2015.

S. Khoshfetrat Pakazad, A. Hansson, M. S. Andersen, and I. Nielsen. “Distributed primal-dual interior-point methods for solving tree-structured coupled convex problems using message-passing”, *Optimization Methods and Software*, DOI:10.1080/10556788.2016.1213839, 2016