Adaptive Control

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Adaptive Control

1. Introduction
2. Model Reference Adaptive Control
3. Recursive Least Squares Estimation
4. The Self-Tuning Regulator
5. Real Adaptive Controllers
6. Summary

Introduction

Adapt to adjust to a specified use or situation

Tune to adjust for proper response

Autonomous independence, self-governing

Learn to acquire knowledge or skill by study, instruction or experience

Reason the intellectual process of seeking truth or knowledge by infering from either fact of logic

Intelligence the capacity to acquire and apply knowledge

In Automatic Control

- Automatic tuning - tuning on demand
- Gain scheduling
- Adaptation - continuous adjustment

Parameter Variations

- Robust control

  Find a control law that is insensitive to parameter variations
- Gain scheduling

  Measure variable that is well correlated with the parameter variations and change controller parameters
- Adaptive control

  Design a controller that can adapt to parameter variations
- Many different schemes

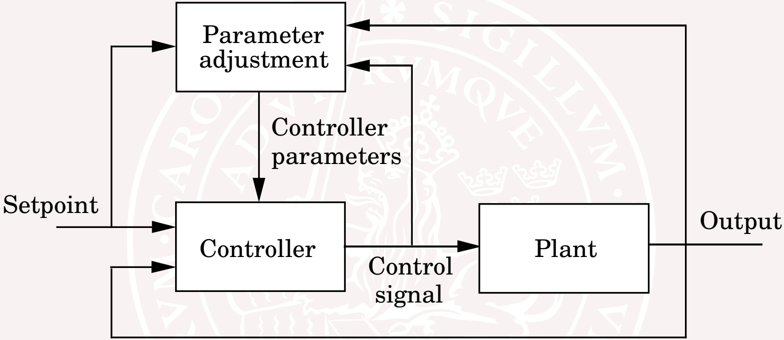
  Model reference adaptive control
  The self-tuning regulator
  L_1 adaptive control (later in LCCC)
- Dual control

  Control should be directing as well as investigating!

A Brief History of Adaptive Control

- Early work driven adaptive flight control 1950-1970.
  - The brave period: Develop an idea, hack a system and make flight tests.
  - Several adaptive schemes emerged no analysis
  - Disasters in flight tests
- Emergence of adaptive theory 1970-1990
  - Model reference adaptive control emerged from stability theory
  - The self tuning regulator emerged from stochastic control theory
- Microprocessor based products 1980 – Novatune
  - Auto-tuners for PID control 1980
- Robustness 1990
- L1-adaptive control - Flight control 2010

Adaptive Schemes



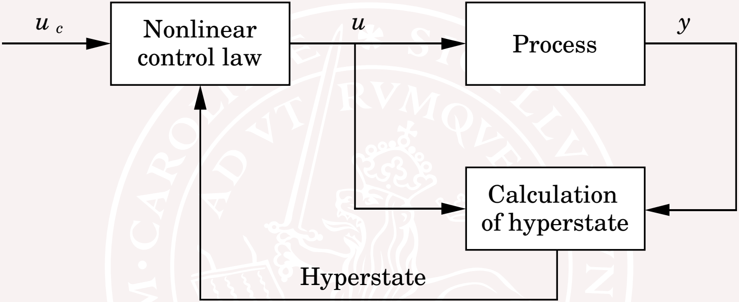
Two loops

- Regular feedback loop
- Parameter adjustment loop

Schemes

- Model Reference Adaptive Control MRAS
- Self-tuning Regulator STR
- Dual Control

Dual Control - Feldbaum



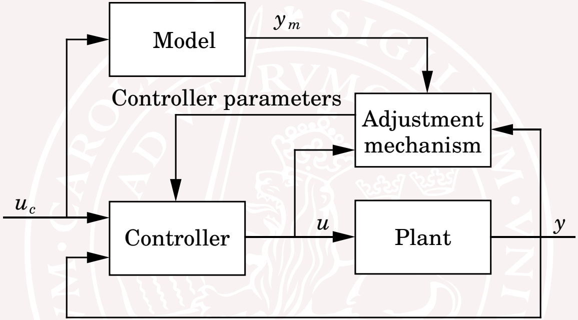
- No certainty equivalence
- Control should be directing as well as investigating!
- Intentional perturbation to obtain better information
- Conceptually very interesting
- Unfortunately very complicated - state is conditional probability distribution of “states” and parameters

Helmerson KJA, worth a second look?

Adaptive Control

1. Introduction
2. Model Reference Adaptive Control
   - Model reference - command signal following
   - The MIT rule -sensitivity derivatives
   - Rules derived from Lyapunov theory
   - Various fixes
3. Recursive Least Squares Estimation
4. The Self-Tuning Regulator
5. Real Adaptive Controllers
6. Summary

Model Reference Adaptive Control



Linear feedback from $e = y - y_m$ is not adequate for parameter adjustment!

The MIT rule

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta}$$

Many other versions

An Example

Process

$$\frac{dy}{dt} = -ay + bu$$

Model

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c$$

Controller

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t)$$

ideal parameters

$$\theta_1 = \theta_1^0 = \frac{b_m}{b}$$

$$\theta_2 = \theta_2^0 = \frac{a_m - a}{b}$$

MIT Rule

The error

$$e = y - y_m, \qquad y = \frac{b\theta_1}{p + a + b\theta_2} u_c \qquad p = \frac{d\,xt}{dt}$$

$$\frac{\partial e}{\partial \theta_1} = \frac{b}{p + a + b\theta_2} u_c$$

$$\frac{\partial e}{\partial \theta_2} = -\frac{b^2\theta_1}{(p + a + b\theta_2)^2} u_c = -\frac{b}{p + a + b\theta_2} y$$

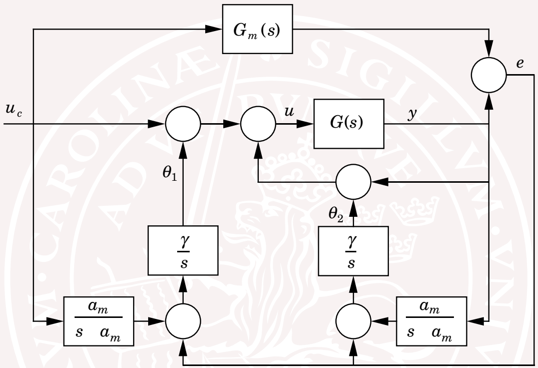
Approximate

$$p + a + b\theta_2 \approx p + a_m$$

Hence

$$\frac{d\theta_1}{dt} = -\gamma \left(\frac{a_m}{p + a_m} u_c \right) e, \qquad \frac{d\theta_2}{dt} = \gamma \left(\frac{a_m}{p + a_m} y \right) e$$

Block Diagram

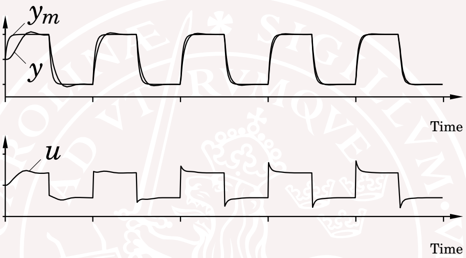


$$\frac{d\theta_1}{dt} = -\gamma \left(\frac{a_m}{p + a_m} u_c \right) e, \qquad \frac{d\theta_2}{dt} = \gamma \left(\frac{a_m}{p + a_m} y \right) e$$

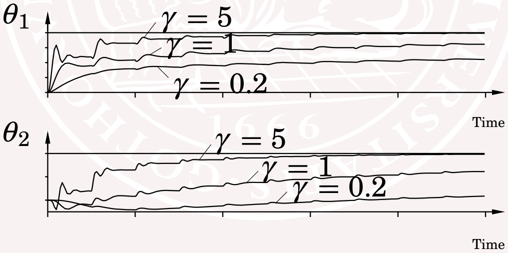
Example $a = 1$, $b = 0.5$, $a_m = b_m = 2$.

Simulation

Input and output



Parameters



Adaptation Law from Lyapunov Theory

- The idea
  - Determine a controller structure
  - Derive the Error Equation
  - Find a Lyapunov function
  - Determine an adaptation law
- A first order system
- State feedback
- Output feedback
  - Passivity
  - Error augmentation>

First Order System

Process model

$$\frac{dy}{dt} = -ay + bu$$

Desired response

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c$$

Controller

$$u = \theta_1 u_c - \theta_2 y$$

The error

$$e = y - y_m$$

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)\, u_c$$

Candidate for Lyapunov function

$$V\,(e, \theta_1, \theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}\,(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}\,(b\theta_1 - b_m)^2\right)$$

Derivative of Lyapunov Function

$$V(e, \theta_1, \theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)^2\right)$$

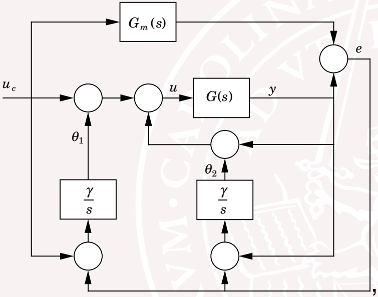
Derivative of Lyapunov function

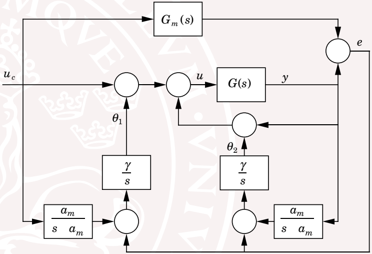
$$\begin{aligned}\frac{dV}{dt} &= e\frac{de}{dt} + \frac{1}{\gamma}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\gamma}(b\theta_1 - b_m)\frac{d\theta_1}{dt} \\ &= -a_m e^2 + \frac{1}{\gamma}(b\theta_2 + a - a_m)\left(\frac{d\theta_2}{dt} - \gamma y e\right) \\ &\quad + \frac{1}{\gamma}(b\theta_1 - b_m)\left(\frac{d\theta_1}{dt} + \gamma u_c e\right)\end{aligned}$$

Adaptation law

$$\frac{d\theta_1}{dt} = -\gamma u_c e, \qquad \frac{d\theta_2}{dt} = \gamma y e$$

Lyapunov (left) vs MIT Rule (right)

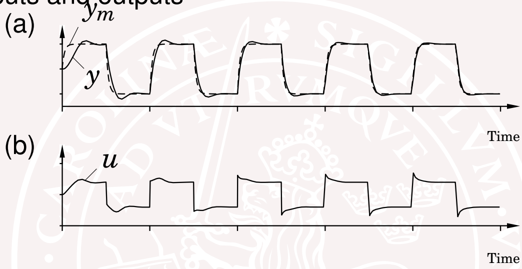




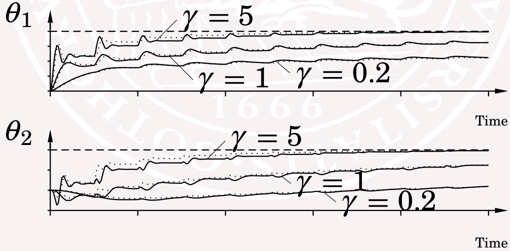
Do the filters matter?

Simulation

Process inputs and outputs



Parameters



State Feedback

$$\frac{dx}{dt} = Ax + Bu$$

Desired response to command signals

$$\frac{dx_m}{dt} = A_m x_m + B_m u_c$$

Control law

$$u = Mu_c - Lx$$

The closed-loop system

$$\frac{dx}{dt} = (A - BL)x + BMu_c = A_c(\theta)x + B_c(\theta)u_c$$

Parametrization

$$A_c(\theta^0) = A_m, \quad B_c(\theta^0) = B_m$$

Compatibility conditions

The Error Equation

$$\frac{dx}{dt} = Ax + Bu$$

Desired response

$$\frac{dx_m}{dt} = A_m x_m + B_m u_c$$

Control law

$$u = M u_c - Lx$$

Error

$$e = x - x_m$$

$$\frac{de}{dt} = \frac{dx}{dt} - \frac{dx_m}{dt} = Ax + Bu - A_m x_m - B_m u_c$$

Hence

$$\begin{aligned}\frac{de}{dt} &= A_m e + (A - A_m - BL)\,x + (BM - B_m)\,u_c \\ &= A_m e + (A_c(\theta) - A_m)\,x + (B_c(\theta) - B_m)\,u_c\end{aligned}$$

The Lyapunov Function

The error equation

$$\frac{de}{dt} = A_m e + \Psi \left(\theta - \theta^0\right)$$

Try

$$V(e,\theta) = \frac{1}{2}\left(\gamma e^T P e + (\theta - \theta^0)^T(\theta - \theta^0)\right)$$

$$\begin{aligned}\frac{dV}{dt} &= -\frac{\gamma}{2}\, e^T Q e + \gamma(\theta - \theta^0)\Psi^T P e + (\theta - \theta^0)^T \frac{d\theta}{dt} \\ &= -\frac{\gamma}{2}\, e^T Q e + (\theta - \theta^0)^T \left(\frac{d\theta}{dt} + \gamma \Psi^T P e\right)\end{aligned}$$

where Q positive definite and

$$A_m^T P + P A_m = -Q$$

Adaptation law and derivative of Lyapunov function

$$\frac{d\theta}{dt} = -\gamma \Psi^T P e, \qquad \frac{dV}{dt} = -\frac{\gamma}{2}\, e^T Q e$$

Extensions

- Output feedback
- Kalman Yakubovich Lemma - Strictly positive real systems
- Various tricks
- Augmented error
- Error normalization $\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta}$
- L1 Adaptive control

The Least Squares Method

The problem: The Orbit of Ceres

The problem solver: Karl Friedrich Gauss

The principle: *Therefore, that will be the most probable system of values of the unknown quantities, in which the sum of the squares of the differences between the observed and computed values, multiplied by numbers that measure the degree of precision, is a minimum.*

In conclusion, the principle that the sum of the squares of the differences between the observed and computed quantities must be a minimum, may be considered independently of the calculus of probabilities.

An observation: Other criteria could be used. *But of all these principles ours is the most simple; by the others we should be led into the most complicated calculations.*

The Book

THEORIA

MOTVS CORPORVM

COELESTIVM

IN

SECTIONIBVS CONICIS SOLEM AMBIENTIVM

AVCTORE

CAROLO FRIDERICO GAVSS.

HAMBVRGI SVMTIBVS FRID. PERTHES ET I. H. BESSER

1809.

Recursive Least Squares

$$
\begin{aligned}
y_{t+1} &= -a_1 y_t - a_2 y_{t-1} + \cdots + b_1 u_t + \cdots + e_{t+1} \\
&= \varphi_t^T \theta + e_{t+1} \\
\hat{\theta}_t &= \hat{\theta}_{t-1} + K_t (y_t - \varphi_t \hat{\theta}_{t-1}) \\
K_t &= P_{t-1} \varphi_t (\lambda + \varphi_t^T P_{t-1} \varphi_t)^{-1} = P_t \varphi_t
\end{aligned}
$$

- Many versions: directional forgetting, resetting, ...
- Square-root filtering (good numerics!)

Persistent Excitation PE

Introduce

$$c(k) = \lim_{t\to\infty} \frac{1}{t} \sum_{i=1}^{t} u(i)u(i-k)$$

A signal u is called *persistently exciting* (PE) of order n if the matrix C_n is positive definite.

$$C_n = \begin{pmatrix} c(0) & c(1) & \dots & c(n-1) \\ c(1) & c(0) & \dots & c(n-2) \\ \vdots & & & \\ c(n-1) & c(n-2) & \dots & c(0) \end{pmatrix}$$

A signal u is persistently exciting of order n if and only if

$$U = \lim_{t\to\infty} \frac{1}{t} \sum_{k=1}^{t} \left(A(q)u(k)\right)^2 > 0$$

for all nonzero polynomials A of degree $n-1$ or less.

Persistent Excitation - Examples

- A step is PE of order 1

$$(q-1)u(t) = 0$$

- A sinusoid is PE of order 2

$$(q^2 - 2q\cos\omega h + 1)u(t) = 0$$

- White noise
- PRBS
- Physical meaning
- Mathematical meaning

Lack of Identifiability due to Feedback

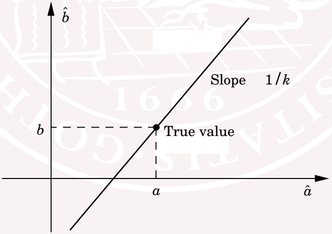
$$y(t) = ay(t-1) + bu(t-1) + e(t), \qquad u(t) = -ky(t)$$

Multiply by α and add, hence

$$y(t) = (a + \alpha k)y(t-1) + (b + \alpha)u(t-1) + e(t)$$

Same I/O relation for all $\hat{a}$ and $\hat{b}$ such that

$$\hat{a} = a + \alpha k, \qquad \hat{b} = b + \alpha$$



Lack of Identifiability due to Feedback

Consider for example the standard feedback loop. **add fig** If the only perturbation on the system is the signal d we have

$$Y(s) = \frac{P(s)}{1 + P(s)C(s)} d, \qquad U(s) = -\frac{C(s)P(s)}{1 + P(s)C(s)} d,$$

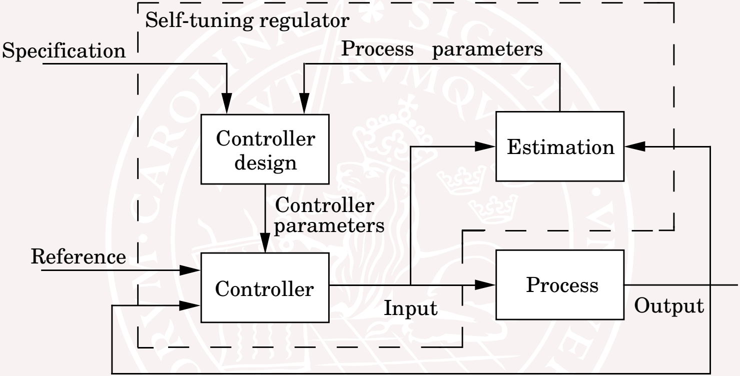
and it thus follows that $Y(s) = -\frac{1}{C(s)} U(s)$ any attempt to find a model relating u and y will thus result in the negative inverse of the controller transfer function. However, if $d = 0$ we have instead

$$Y(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} (F(s)R(s) - N(s))$$
$$U(s) = \frac{C(s)}{1 + P(s)C(s)} (F(s)R(s) - N(s)d,$$

hence $Y(s) = P(s)U(s)$ and the process model can indeed be estimated.

Adaptive Control

The Self-Tuning Regulator



- Certainty Equivalence - Design as if the estimates were correct (Simon)
- Many control and estimation schemes

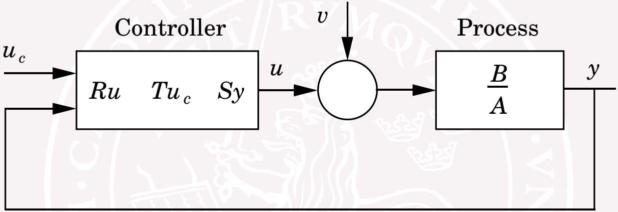
The Self-Tuning Regulator

- Estimate parameters recursively
- Compute control law from estimated parameters
- Apply control signal
- Many different choices
  - Model structure
  - Parameterization
  - Criterion
- A simple choice:
  - Design an adaptive minimum variance controller.

Command Signal Following

Process and controller

$$Ay(t) = B\left(u(t) + v(t)\right), \qquad Ru(t) = Tu_c(t) - Sy(t)$$



Closed loop system

$$y(t) = \frac{BT}{AR+BS}u_c(t) + \frac{BR}{AR+BS}v(t)$$

$$u(t) = \frac{AT}{AR+BS}u_c(t) - \frac{BS}{AR+BS}v(t)$$

Closed loop characteristic polynomial

$$AR + BS = A_c$$

Closed loop response

$$y(t) = \frac{BT}{AR + BS} u_c(t)$$

Desired response

$$A_m y_m(t) = B_m u_c(t)$$

Perfect model following

$$\frac{BT}{AR + BS} = \frac{BT}{A_c} = \frac{B_m}{A_m}$$

Avoid cancelation of unstable process zeros :$B = B^+ B^-$

$$B_m = B^- B_m', \qquad A_c = A_o A_m B^+, \qquad R = R' B^+$$

Hence

$$AR' + B^- S = A_o A_m = A_c', \qquad T = A_o B_m'$$

Example

$$G(s) = \frac{1}{s(s+1)}$$

Sampling with $h = 0.5$

$$H(q) = \frac{b_0 q + b_1}{q^2 + a_1 q + a_2} = \frac{0.1065q + 0.0902}{q^2 - 1.6065q + 0.6065}$$

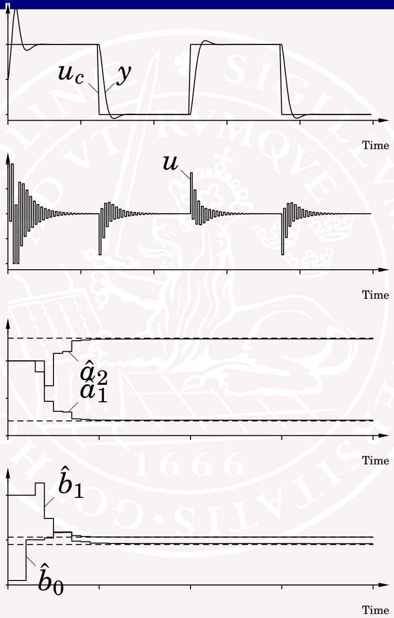
$$\frac{B_m(q)}{A_m(q)} = \frac{b_{m0} q}{q^2 + a_{m1} q + a_{m2}} = \frac{0.1761q}{q^2 - 1.3205q + 0.4966}$$

$$B^+(q) = q + b_1/b_0, \qquad B^-(q) = b_0, \qquad B'_m(q) = b_{m0} q / b_0$$

Choose $A_o = 1$

$$u(t) + r_1 u(t-1) = t_0 u_c(t) - s_0 y(t) - s_1 y(t-1)$$

Example



Redesign with no Cancellation

$$H(q) = \frac{b_0 q + b_1}{q^2 + a_1 q + a_2} = \frac{0.1065q + 0.0902}{q^2 - 1.6065q + 0.6065}$$

We have $B^+ = 1$ and $B^- = B = b_0 q + b_1$

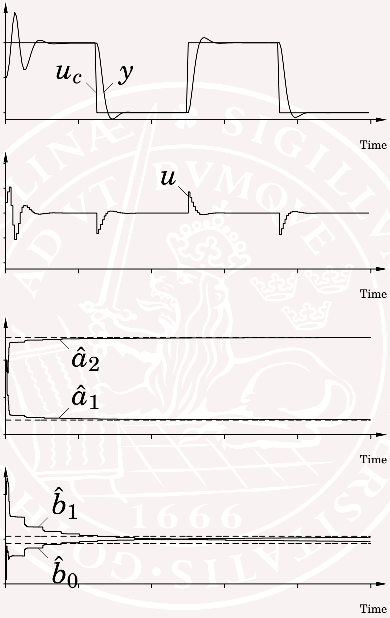
$$H_m(q) = \beta \frac{b_0 q + b_1}{q^2 + a_{m1} q + a_{m2}} = \frac{b_{m0} q + b_{m1}}{q^2 + a_{m1} q + a_{m2}}$$

Diophantine equation

$$\begin{aligned}(q^2 + a_1 q + a_2)(q + r_1) + (b_0 q + b_1)(s_0 q + s_1) \\ = (q^2 + a_{m1} q + a_{m2})(q + a_o)\end{aligned}$$

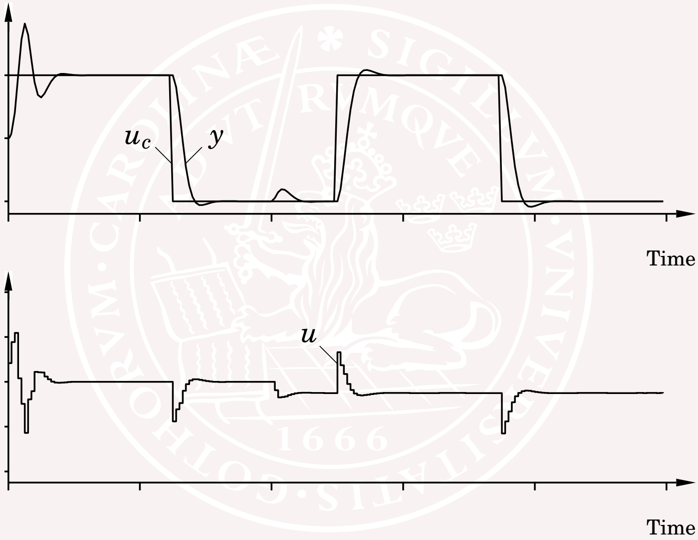
Control law

$$u(t) + r_1 u(t-1) = t_0 u_c(t) - s_0 y(t) - s_1 y(t-1)$$



u_c
y
Time
u
Time
$\hat{a}_2$
$\hat{a}_1$
Time
$\hat{b}_1$
$\hat{b}_0$
Time

Load Disturbances



Direct and Indirect STR

Process and controller

$$Ay = Bu, \qquad Ru = Tu_c - Sy$$

Direct: Estimate A and B, compute R and S from

$$AR + BS = A_oA_m$$

Indirect: Form

$$A_oA_my = ARy + BSy = BRu + BSy = B(Ru + Sy)$$

Estimate R and S by least squares from

$$A_oA_my = B(Ru + Sy)$$

Combine with estimate of B from the direct method and use control law

$$\hat{R}u = A_ou_c - \hat{S}y$$

Adaptation of Feedforward Gains

Process model

$$Ay = Bu + Cv$$

where v is a measured disturbance.

Estimate R, S and Q by least squares from

$$A_o A_m y = B(Ru - Sy - Qv)$$

Use control law

$$\hat{R}u = A_o u_c - \hat{S}y - \hat{Q}v$$

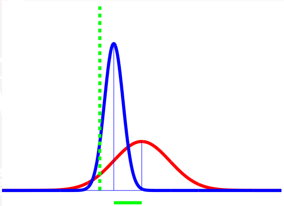
STR and Minimum Variance Control - Björn

$$y_{t+1} + a y_t = b u_t + e_{t+1} + c e_t$$

$$u_t = \frac{1}{b}(a y_t - c e_t) = \frac{a-c}{b} y_t$$

In general $A(q)_t y = B(q) u_t + C(q) e_t$

$$u_t = \frac{a(q) - c(q)}{b(q)} (y_t - r_t)$$



The self-tuning controller

$$y_t = \beta(u_t - \theta y_t) + \epsilon$$

$$u_t = \mathrm{sat}\Big(\hat{\theta}(y_t - r_t)\Big)$$

Estimate θ by least squares for fixed β, $0.5 < \beta/b < \infty$, $B(z)$ stable + order conditions. Local stability: real part of $C(z)$ positive for all zeros of $B(z)$

An Example

Consider a process governed by

$$y(t+1) + ay(t) = u(t) + e(t+1)$$

Assume that we would like to keep the output as close to zero as possible in the least squares sense. Estimate parameter a by least squares and use the control law $u = \hat{a}y(t)$. The estimate is given by the normal equation

$$y(t+1) + \hat{a}y(t) = u(t)$$

by least squares. The normal equation is

$$\sum_{k=1}^{t-1} y^2(k)\hat{a}(t) = \sum_{k=1}^{t-1} (u(k) - y(k+1))y(k)$$

An Example ...

Using recursive computations of the estimate the control algorithm becomes

$$
\begin{aligned}
u(t) &= \hat{a}(t)y(t) \\
\hat{a}(t) &= \hat{a}(t-1) + K(t)(y(t) - u(t-1) + \hat{a}(t-1)y(t-1)) \\
K(t) &= P(t)y(t-1) \\
P(t) &= P(t-1) - \frac{P^2(t-1)y^2(t-1)}{1 + y^2(t-1)P(t-1)}
\end{aligned}
$$

Clearly a nonlinear time-varying controller. The algorithm behaves as expected when applied to a system with the right model structure. What happens if it is applied to the system

$$
y(t+1) + ay(t) = bu(t) + e(t+1) + ce(t)
$$

Does the algorithm have some general properties?

An Example ...

Consider the system

$$y(t+1) - 0,9y(t) = 3u(t) + e(t+1) - 0.3e(t)$$

The minimum variance controller is

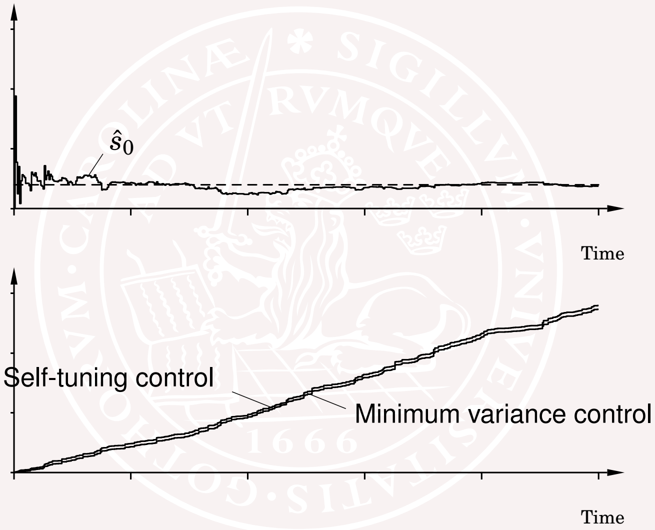
$$u(t) = -\frac{c-a}{b}y(t) = -0.2y(t)$$

Control the process by a self-tuner that estimates parameters in

$$y(t+1) + ay(t) = u(t) + e(t+1)$$

and use the control algorithm $u = \hat{a}y(t)$. Notice that the model structure is wrong!

Example ...



$\hat{s}_0$
Time
Self-tuning control
Minimum variance control
Time

Example ...

Consider a process governed by

$$y(t+1) + ay(t) = bu(t) + e(t+1) + ce(t)$$

Estimate the parameter a in

$$y(t+1) + ay(t) = u(t)$$

by least squares. The normal equation is

$$\sum_{k=1}^{t-1} y^2(k)\hat{a}(t) = \sum_{k=1}^{t-1} (u(k) - y(k+1))y(k)$$

The control law is

$$u(t) = \hat{a}(t)y(t)$$

Example ...

$$\sum_{k=1}^{t-1} y^2(k)\hat{a}(t) = \sum_{k=1}^{t-1}(u(k) - y(k+1))y(k)$$
$$= \sum_{k=1}^{t-1}(\hat{a}(k)y(k) - y(k+1))y(k)$$

$$\frac{1}{t}\sum_{k=1}^{t-1} y(k+1)y(k) = \frac{1}{t}\sum_{k=1}^{t-1}(\hat{a}(k) - \hat{a}(t))y^2(k)$$

If $y(k)$ is bounded in the mean square and if the estimate converges we find that the closed loop system has the property

$$\lim_{t\to\infty} \frac{1}{t}\sum_{k=1}^{t-1} y(k+1)y(k) = 0$$

Example ...

If the parameters converges and the signals remain mean square bounded the simple self tuner drives the correlation of the output

$$\lim_{t\to\infty} \frac{1}{t} \sum_{k=1}^{t-1} y(k+1)y(k) = r_y(1) = 0$$

to zero.

Compare with a controller with integral action which drives the error to zero if the closed loop system is stable.

What does the condition $r_y(1) = 0$ imply?

Example ...

Consider the system

$$y(t+1) + ay(t) = bu(t) + e(t+1) + ce(t)$$

with the feedback

$$u(t) = -ky(t)$$

The closed loop system is

$$y(t) = \frac{q+c}{q+a+bk} e(t) = e(t) + \frac{c-a+bk}{q+a+bk} e(t-1)$$

The condition $r_y(1) = 0$ implies that $c - a + bk = 0$ and that $r_y(\tau) = 0$ for all $\tau \neq 0$.

The simple self-tuner converges to the minimum variance controller

Adaptive Control

ASEA Novatune

NOVATUNE

Process Control with Adaptive Controllers

NOVATUNE

ASEA

ASEA Novatune

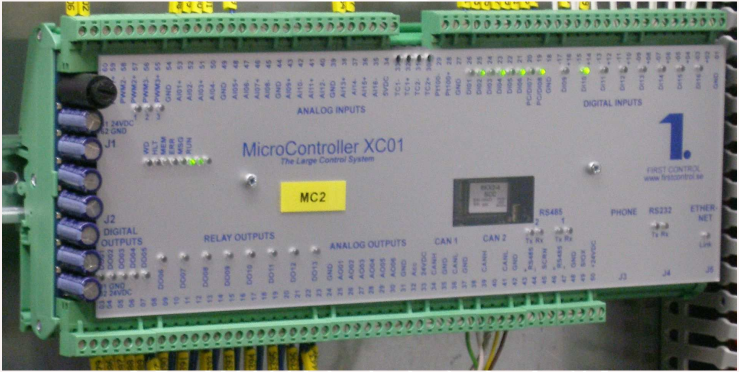
Made in Sweden

At the beginning of the 70s, the basic theory of self-tuning, adaptive control was developed by a group around professor K-J Åström at the Lund Institute of Technology in Sweden. At the same time ASEA initiated the first industrial installations. Today this technique is well-known throughout the world.

ASEA NOVATUNE is an instrumentation system based on adaptive control.

Adaptation of feedforward from measured disturbances

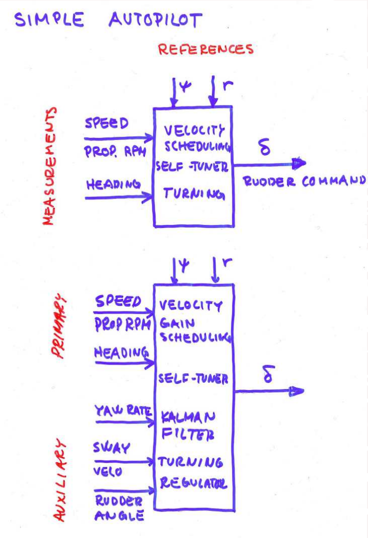
First Control - Gunnar Bengtsson

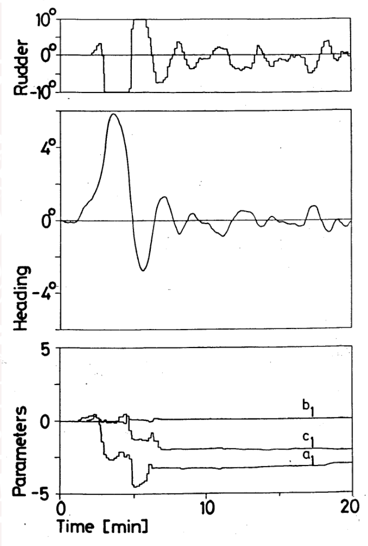


Winner of IEEE CSS Control Technology Award

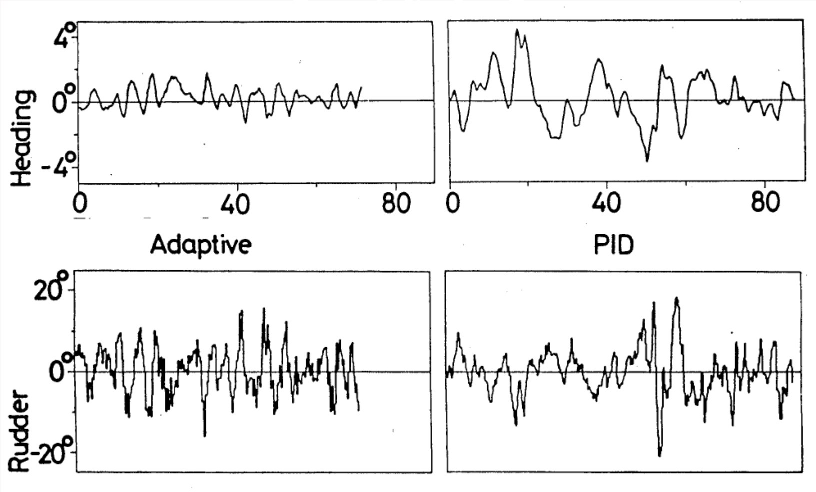
1. FIRST CONTROL SYSTEMS AB

Ship Steering - Clas Källström





Ship Steering - Performance



Steermaster

NORTHROP GRUMMAN

Relay Auto-Tuner Tore

- One-button tuning
- Automatic generation of gain schedules
- Adaptation of feedback and feedforward gains
- Many versions
  - Single loop controllers
  - DCS systems
- Robust
- Excellent industrial experience
- Large numbers

Adaptive Control

Summary

- Adaptive control has had a turbulent history
- Adaptive systems are now reasonably well understood
- They are nonlinear and not trivial to analyse and design
- Interesting ideas
  - Stability theory, passivity
  - Chaotic behavior
  - Averaging based on the assumption that parameters are slow
  - Lennart's differential equations
- A large field we have just given a sketch
- Important issues still unresolved
- There are a number of adaptive systems running in industry
- Initialization, safety nets and guards
- Excitation and load disturbances