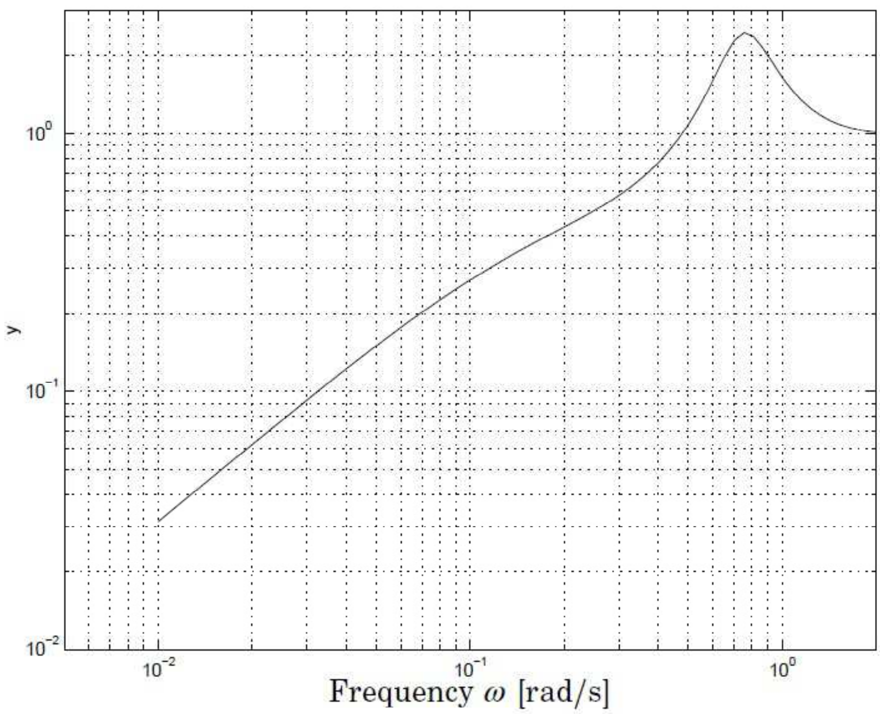
Exercise Session 1

1. To evaluate a controlled system the maximum values of the sensitivity function and the complementary sensitivity functions have been computed giving

$$\max_{\omega} |S(i\omega)| = 2.45, \quad \max_{\omega} |T(i\omega)| = 1.70$$

Use these numbers to estimate the largest amplification of disturbances that may occur. Also provide an estimate of the precision in the transfer function required for the closed loop system to be stable.

The figure below shows the magnitude of the sensitivity function of the system.



Give a frequency range where disturbances are reduced by a factor of 10. Give the frequency range where the feedback increases the disturbances. What is the frequency where the increase is largest.

Before installing the controller the output has been measured. The meaured data could be approximately described by the following function

$$y_{open\ loop} \approx 20 \sin 0.01t + 10 \sin 0.1t + \sin 0.7t$$

The root mean square fluctuation of the output in open loop is thus

$$\sqrt{20^2 + 10^2 + 1^2} \cdot 1/\sqrt{2} = \approx 22.4/\sqrt{2}$$

Make an estimate of the mean square fluctuation of the output under closed loop control under the assumption that the disturbances have the same properties as when the measurements were made.

2. Consider a closed loop system where the maximum sensitivity is M_s. Prove the following inequalities for the gain and phase margins

$$g_m \geq \frac{M_s}{M_s - 1} \qquad \phi_m \geq 2 \arcsin \frac{1}{2M_s}$$

3. Consider a process with the transfer function

$$P(s) = e^{-\sqrt{sT}}.$$

Determine the critical gain under proportional (P) and integral (I) control.

4. Consider a system with transfer function

$$G(s) = \frac{1}{(s+1)^3}.$$

Assume a disturbance $l = \sin \omega t$ acts on the process input. Investigate the error obtained with a P controller with $k = 1.2$ and PI controller with $k = 1.2$ and $T_i = 1.8$. Then construct a controller so that the error due to the sinusoidal disturbance is less than 0.001! You can set $\omega = 2\pi/T$ with $T = 30$. If you have time you can also investigate how small T can be.

5. Consider the following system with a slow process pole $s = -0.1$

$$G(s) = \frac{1}{10s + 1}$$

controlled with the controller

$$C(s) = \frac{10(10s + 1)}{s}$$

cancelling the slow pole and putting the closed loop pole in $s = -10$. Is fast control achieved? Compute the Gang of Four and study the response of an input load disturbance.

6. On the exercise we will also familiarize ourselves with the Flexible Servo Benchmark problem.