Gain Scheduling

Bo Bernhardsson and Karl Johan Åström

Department of Automatic Control LTH,
Lund University

Gain Scheduling

- What is gain scheduling ?
- How to find schedules ?
- Applications
- What can go wrong ?
- Some theoretical results
- LPV design via LMIs
- Conclusions

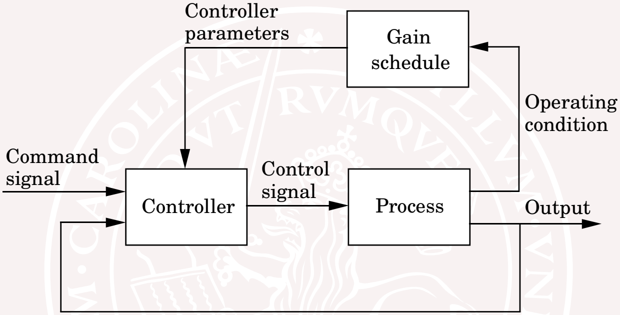
To read:

Leith & Leithead, Survey of Gain-Scheduling Analysis & Design

To try out:

Matlab - Gain Scheduling

Gain Scheduling



Example of scheduling variables

- Production rate
- Machine speed, e.g. DVD player
- Mach number and dynamic pressure

How to Find Schedules?

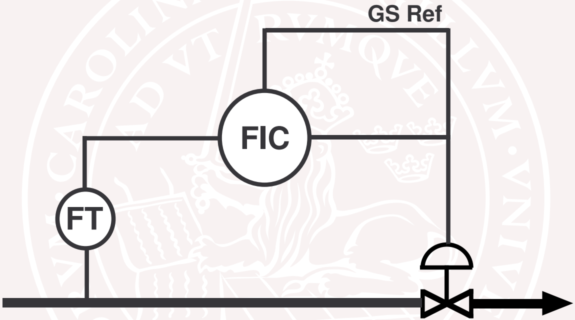
Select scheduling variables: Variable(s) should reflect changes in system dynamics.

Make (linear) control design for different operating conditions: For instance with automatic tuning

Use "closest" control design, or interpolate: Many ad-hoc or theoretically motivated methods exist

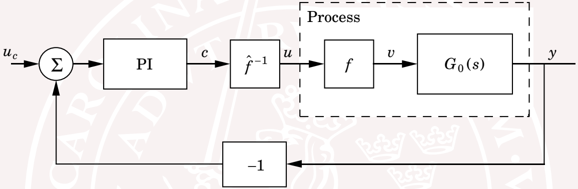
Verify performance: Simulations. Some methods exist that guarantee performance; usually conservative though

Scheduling on controller output

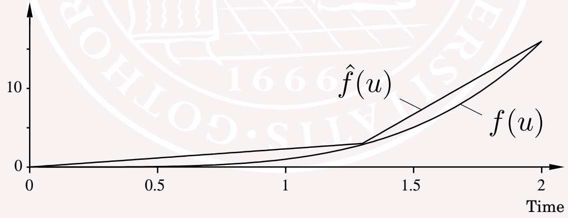


Nonlinear Valve

A typical process control loop



Valve characteristics and a crude approximation

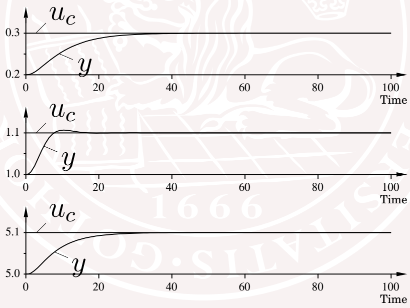


Results

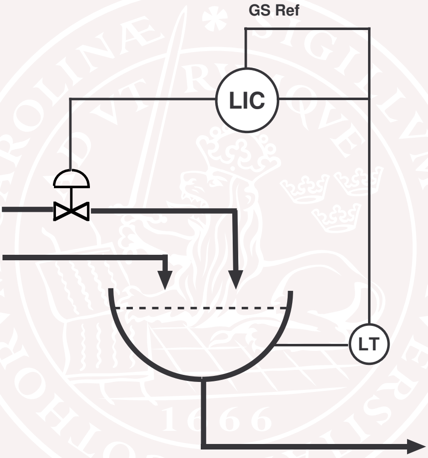
Without gain scheduling

Loop is either too slow or unstable

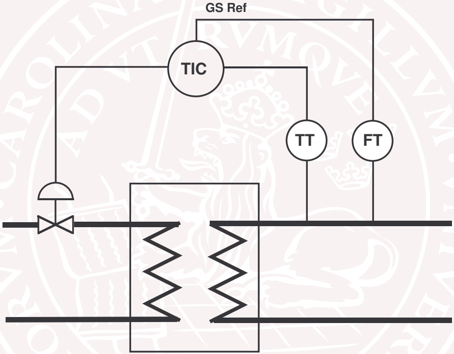
With gain scheduling



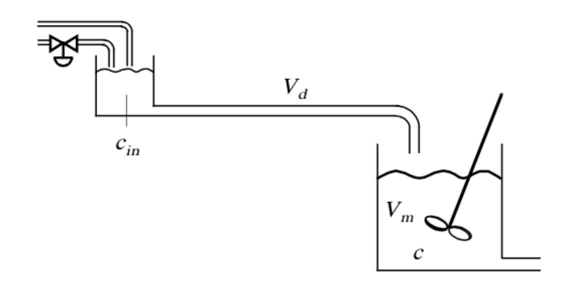
Schedule on Process Variable



Schedule on External Variable

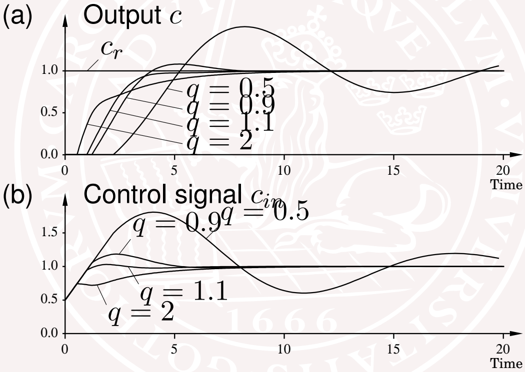


Concentration Control



Concentration Control

Performance with changing flow V_d



Variable Sampling Rate

Process model

$$G(s) = \frac{1}{1+sT}e^{-s\tau}, \quad T = \frac{V_m}{q}, \quad \frac{V_d}{q}$$

Sample system with period

$$h = \frac{V_d}{nq}$$

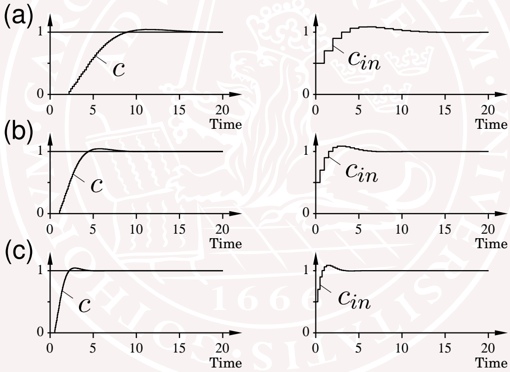
Sampled model becomes linear ”time”-invariant

$$c(kh+h) = ac(kh) + (1-a)u(kh-nh), \qquad a = e^{-qh/V_m} = e^{-V_d/(nV_m)}$$

Sampled equation does not depend on q !!

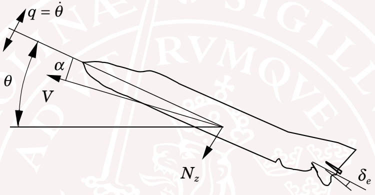
Results

Digital control with $h = 1/(2q)$.
The flows are: (a) $q = 0.5$; (b) $q = 1$; (c) $q = 2$

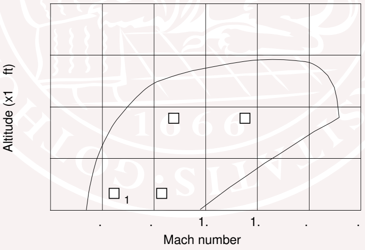


Flight control

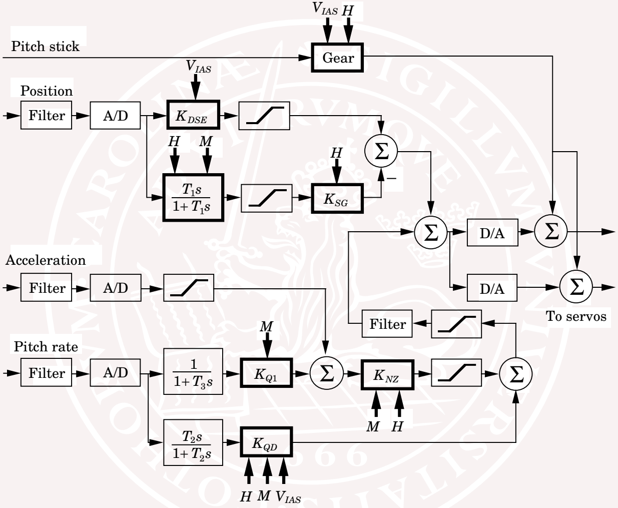
Pitch dynamics



Operating conditions

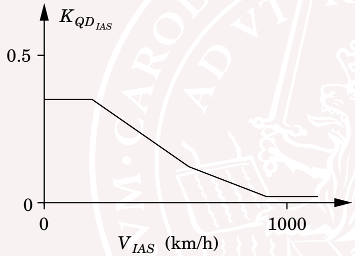


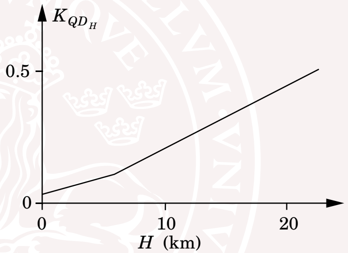
The Pitch Control Channel



Many scheduling variables

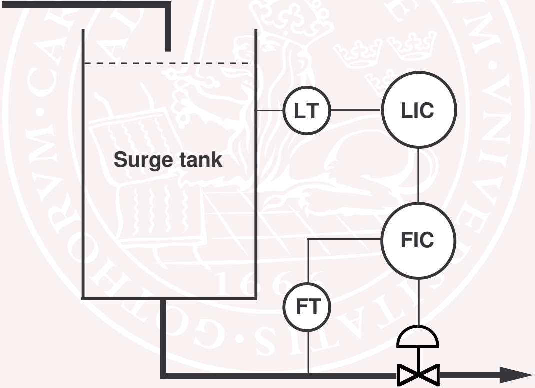
Schedule of K_Q wrt airspeed (IAS) and height (H)



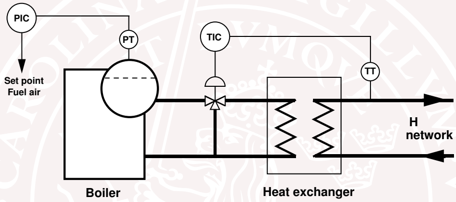


Surge Tank Control

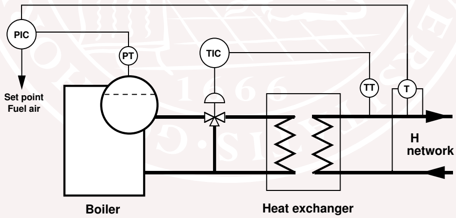
A surge tank is used to smooth flow variations. The is allowed will fluctuate substantially but it is important that the tank does not become empty or that it overflows.



Controller structure before modification



Modified controller structure



What can go wrong?

Most designs are done for the time-frozen system, i.e. as if scheduling parameter θ is constant.

Theory and practice: This will work also when $\theta(t)$ is slowly varying. But can go wrong for fast varying parameters.

Following example is from Shamma and Athans, ACC 1991

Shamma - Athans

Resonant system with varying resonance frequency

$$G_\theta(s)\frac{1}{s} = \frac{1}{s^2 + 0.2s + 1 + 0.5\theta(t)}\frac{1}{s} = C(sI - A(\theta))^{-1}B$$

with $-1 \le \theta(t) \le 1$

Controller design: LQG + integrator on system input, LQG parameters

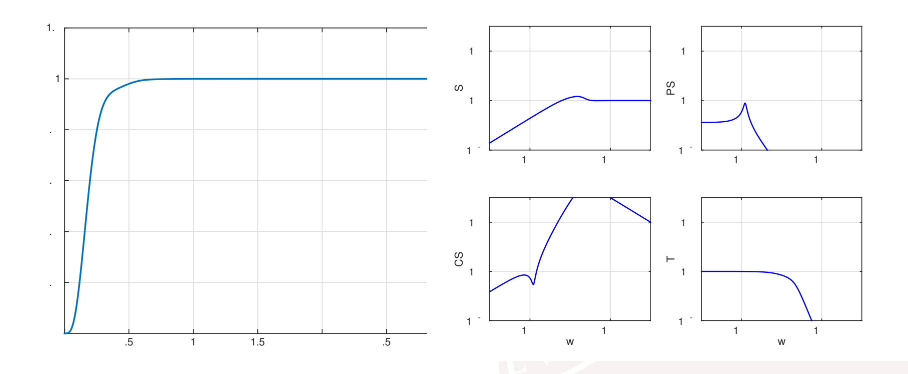
$$Q_{11} = C^T C, \quad Q_{22} = 10^{-8}, \quad R_{11} = B_2(\theta)B_2(\theta)^T, \quad R_{22} = 10^{-2}$$

Gives controller $K_\theta(s)$ with good robustness margins when θ constant

Frozen-system loop-gain $G_\theta(s)K_\theta(s)$ is actually independent of θ (using $B_2(\theta)^T = [1 \;\; 0 \;\; 1 + 0.5\theta]$)

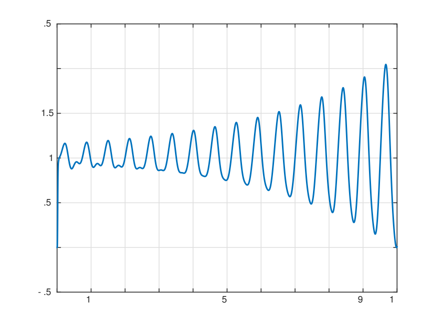
Shamma-Athans

Step response and GOF for any constant θ look fine.
All θ give the same curves



Shamma-Athans

But if $\theta(t) = \cos(2t)$ the system becomes unstable



It can be shown that the open loop LTV system is unstable in this case.

Several theorems show that " if θ varies slowly" performance for the frozen-system analysis is maintained for the true system

- Small-gain theorem
- Lyapunov theory using $V = x^T P x$ or $V = x^T P(\theta) x$

Gain-scheduling design

Several authors have worked with systematic design methods

- Shamma-Athans
- Packard
- Apkarian-Gahinet
- Helmersson
- ...

Gain-scheduling for LPV systems by LMIs

Apkarian, Gahinet (1995) A Convex Characterization of Gain-Scheduled H_∞ Controller

Model assumption

$$\begin{aligned}\dot{x} &= A(\theta(t))x(t) + B(\theta(t))u(t)\\ y &= C(\theta)x(t) + D(\theta(t))u(t)\end{aligned}$$

Controller structure

$$\begin{aligned}\dot{\zeta}(t) &= A_K(\theta(t))\zeta(t) + B_K(\theta(t))y(t)\\ u(t) &= C_K(\theta(t))\zeta(t) + D_K(\theta(t))y(t)\end{aligned}$$

Will assume both process and controller depends on θ via a fractional transformation.

Main Idea

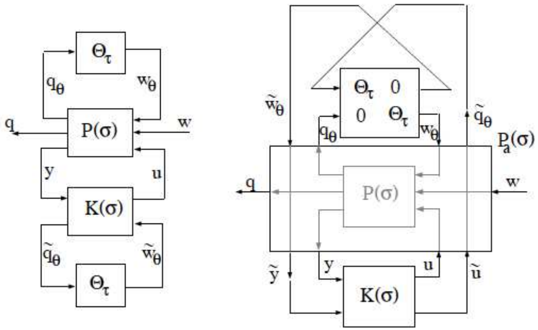


Figure 2.1: a) LPV control structure – b) Transformed structure

$$\begin{pmatrix}\tilde{q}_\theta & q_\theta & q & y & \tilde{w}\end{pmatrix}^T = P_a(s)\begin{pmatrix}\tilde{w}_\theta & w_\theta & w & u & \tilde{u}\end{pmatrix}^T$$

Main Result - a sufficency condition

The closed loop system is stable for all $\theta(t)$ with $\|\theta(t)\| < 1/\gamma$ and the L_2 induced norm from w to q satisfies

$$\max_{\|\theta(t)\|<1/\gamma^2} \|T_{qw}\| < \gamma$$

if there is a scaling matrix L commuting with Θ so that

$$\| \begin{pmatrix} L^{1/2} & 0 \\ 0 & I \end{pmatrix} F_l(P_a, K) \begin{pmatrix} L^{-1/2} & 0 \\ 0 & I \end{pmatrix} \| < \gamma$$

Sufficient, not necessary

The condition can be checked by an LMI, also gives the controller.

If Time Permits

http://se.mathworks.com/help/control/ug/gain-scheduled-control-of-a-chemical-reactor.html

Summary - Gain Scheduling

Very useful technique

- Linearization of nonlinear actuators
- Surge tank control
- Control over wide operating ranges

Requires good models

Issues to consider

- Choice of scheduling variable(s)
- Granularity of tables, interpolation
- Bumpless parameter changes