Handin 1

Bo Bernhardsson, K. J. Åström

Department of Automatic Control LTH,
Lund University

Handin 1 - goals

Get some practice using the Matlab control system toolbox (or similar)

Get started with some control design

Example - Double Integrator

Consider the double integrator

$$y = \frac{1}{s^2} u$$

controlled with state-feedback + Kalman filter

$$u = -K\hat{x} = -K(sI - A + BK + LC)^{-1} Ly$$

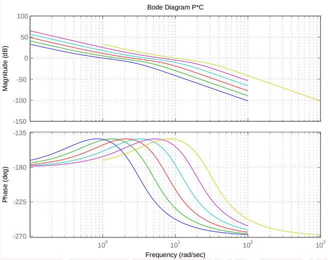
Let's place eigenvalues of

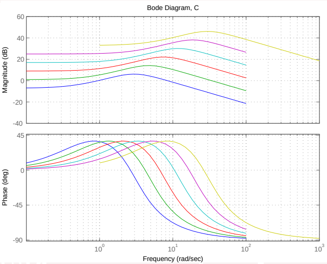
$$A - BK \quad \text{and} \quad A - LC$$

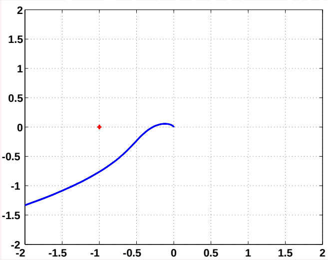
in Butterworth patterns

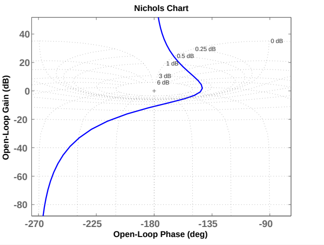
```
>Pm = butter(2,wm,'s')
>K  = place(A,B,Pm)
>Po = butter(2,wo,'s')
>L  = place(A',C',Po)'
```

Results

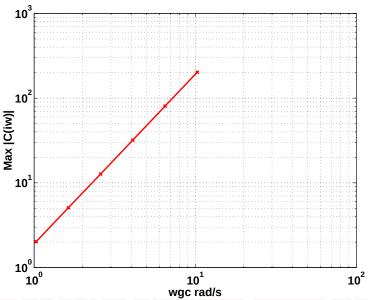

Bode Diagram P*C
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)


Bode Diagram, C
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)




Nichols Chart
Open-Loop Gain (dB)
Open-Loop Phase (deg)
0 dB
0.25 dB
0.5 dB
1 dB
3 dB
6 dB

Results



Trade off between closed loop bandwidth and controller gain

(Hmmmm, why is the slope 2?)

(Advanced hmmmm, why do LQG design with varying control penalty ρ and Butterworth pole placement give the same results? Answer to this later in the course.)

Handin 1A- PI control of 1st order system

First order model: the archetype system

Normalizing input, output and time variables we have

$$G(s) = \frac{1}{s+1}$$

Let's use PI control

$$u = (k_p + k_i/s)(r - y)$$

This gives a 2nd order closed system.

Handin 1A- PI control of 1st order system

Let's design the closed loop polynomial to become

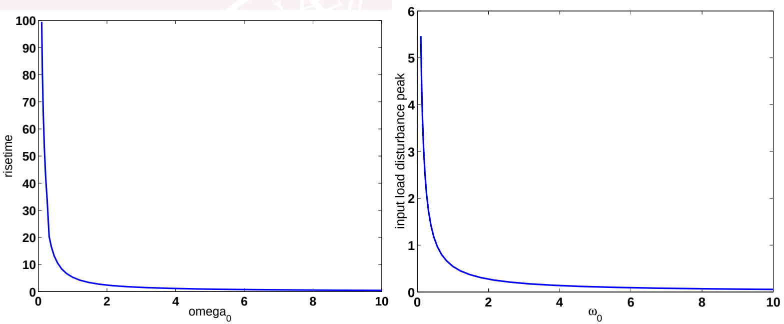
$$s^2 + 2\zeta_0\omega_0 s + \omega_0^2$$

we get

$$\begin{aligned} k_p &= 2\zeta_0\omega_0 - 1 \\ k_i &= \omega_0^2 \end{aligned}$$

Let's assume a slow design is ok, say $\omega_0 = 0.1, \zeta_0 = 0.5$.

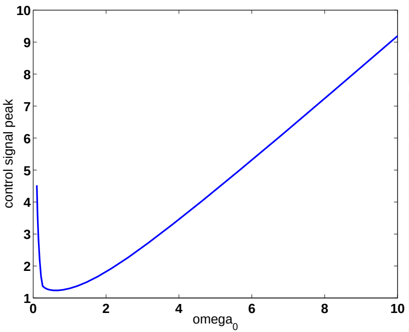
Result - Rise time vs ω_0



Looks as expected.

Let's check the control signal size also.

Step Response - input signal size



Hmm, the behavior when ω_0 is small is rather unexpected.

Let's check the Bode and Nyquist diagrams.

Handin 1

Exercise 1A Verify the previous figures. Use pole placement design to do PI control of the system $1/(s+1)$ for varying ω_0.

Use any method you like to find a PI-controller that achieves good robustness and a gain-crossover frequency $\omega_{gc} = 0.1$, or describe why this is not possible.

Handin 1

Exercise 1B Consider the system $P(s) = \frac{s+1}{s^2}$. Design a controller with pole-placement where the observer poles and the controller poles have $\omega_0 = 10$ and damping ratio $\zeta_0 = 0.707$. Plot the Nyquist curve of the loop transfer function and the Gang of Four for the closed loop systems obtained. Comment on the design.

Handin 1 is due Monday Feb 22, 15.00