Mixed H_∞/H_2-synthesis and Youla-parametrization

Olof Troeng

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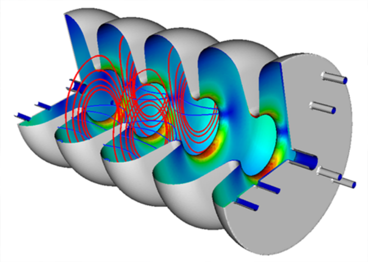
Motivation (1/2)

Control of electric field in accelerator cavity.



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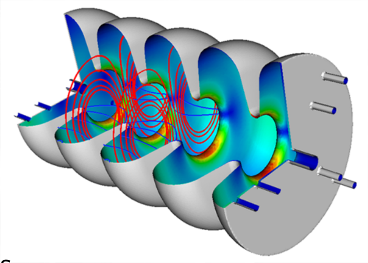

Very simple process

$$P(s) = \frac{1}{1 + sT} \mathrm{e}^{-s\tau},$$

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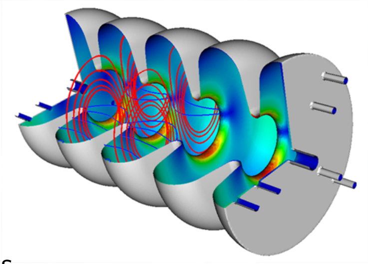

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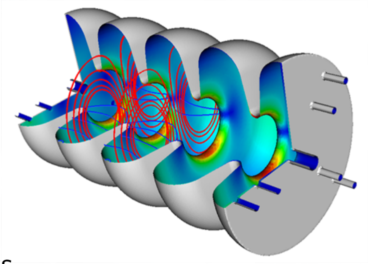

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Inspiration from (Garpinger 2009).

Motivation (2/2)

Want to solve optimization problem

$$
\begin{aligned}
\underset{K}{\text{minimize}} \quad & ||PSH||_2 \\
\text{subject to} \quad & ||S||_\infty \leq M_S. \\
& ||KSN||_2 \leq L_n.
\end{aligned}
$$

where $S = \dfrac{1}{1 + PK}$
H – spectrum of load disturbance
N – spectrum of measurement noise
M_S – sensitivity constraint
L_n – bound on control signal activity due to measurement noise

How to find optimal K?

When the H_∞-constraint is active, the optimal controller *is* infinite dimensional [Megretski] :..(

Much literature in the 90's (including (Bernhardsson 1992)), mostly LMI and Riccatti techniques similar to H_∞ synthesis (Khargonekar et al. 1991; Scherer et al. 1997). Gives subpotimal controller.

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Youla parametrization or Q-parametrization (Boyd et al. 1991; Megretski 2011).

Q-parametrization

Internal stability means stability of

$$\begin{bmatrix} \dfrac{1}{1+PK} & \dfrac{P}{1+PK} \\ \dfrac{K}{1+PK} & \dfrac{PK}{1+PK} \end{bmatrix} \qquad (*)$$

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$$Q = \frac{K}{1+PK}$$

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Then

$$(*) = \begin{bmatrix} 1 - PQ & P(1-PQ) \\ Q & PQ \end{bmatrix}$$

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Introduce

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Then

$$(*) = \begin{bmatrix} 1-PQ & P(1-PQ) \\ Q & PQ \end{bmatrix}$$

and assuming P is stable:

$$\text{stability of } Q \Leftrightarrow \text{stability of } (*)$$

Finite-dimensional parametrization of Q

Restrict Q to finite-dimenional space spanned by $\{Q_k\}$,

$$Q = \sum_{k=1}^{N} \beta_k Q_k$$

Discretize time-domain and frequency-domain constraint over time grid $T = \{\tau_j\}$ and $\Omega = \{\omega_j\}$

For example $||S||_\infty \leq M_S$:

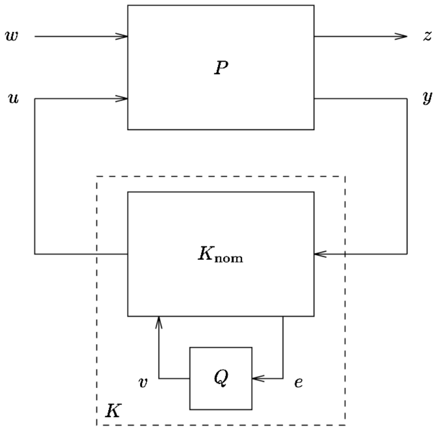
$$|1 - P(\omega_j)Q(\omega_j)| \leq M_S \qquad \forall \omega_j \in \Omega$$

Many useful constraints are convex in Q

- Rise-time
- Settling-time
- Sensitivity bound
- Control signal activity

Can be handled using e.g. `cvx`.

Q-parametrization (unstable P)



Find stabilizing *state observer/state feedback* controller. Augment the stabilized system with Q-parameter in special way (Boyd et al. 1991; Megretski 2011).

Thank you for listening!

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