# Rootlocus

**Gautham**

Department of Automatic Control

## Rootlocus Method (Rotortmetoden)

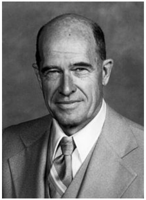
## Plotting of the root locus

# The Rootlocus Method(Rotortmetoden)

## Introduction

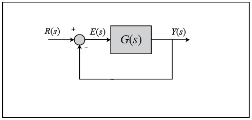
- Graphical method of solving algebraic equations introduced by Walter R.Evans. in 1948.
- Instead of solving equations for fixed values of parameters, the equation is solved for all values of any parameters(or their combination).
- An algebraic equation $A(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + ... + a_n$ can be written as $A(s) = P(s) + Q(s)$
- Now instead of solving the equation $A(s) = P(s) + I.Q(s)$, we solve the equation $A(s) = P(s) + K.Q(s)$ for positive real values of K. Thus the original problem is a more general case of the latter.

Walter R.Evans



# Root locus and Feedback

- The general case for the root locus is solved for cases of K varying from 0 to $\infty$ and this gives the root locus of the general equation.
- Very useful in fb system analysis as the general case is a characteristic equation
- Can be used in a system with one unknown parameter. For example $\frac{1}{s^3+4s^2+Ks+1}$
- Consider a system with unity feedback as shown in the block diagram



# Feedback

- $$G_0(s) = K\frac{Q(s)}{P(s)} \tag{1}$$

$$G(s) = \frac{KQ(s)}{(P(s) + KQ(s))} \tag{2}$$

$$P(s) + KQ(s) \tag{3}$$

- The Closed loop has a characteristic polynomial very similar to the general case equation above.
- Example 1

# Rules for plotting the root locus

- Equation $P(s) + KQ(s)$ can be written in magnitude and phase as

$$argQ(s) - argP(s) = \pi + 2k\pi, k = 1, 2, ... \tag{4}$$

$$K = \frac{|P(s)|}{|Q(s)|} \tag{5}$$

- Thus all points that satisfy s will lie on the root locus. There are some rules/observations that can be used to simplify plotting the root locus

# Rules

1. **Symmetry:** If P(s) and Q(s) have real coefficients
2. **Number of Branches:** Equal to the largest degree of P(s) and Q(s)
3. **Start and End points:** Go from K=0 to K=$\infty$. Or in infinity depending upon the number of poles and zeros
4. **Near the start and end points:** To the right of every point on the root locus, there must be an odd number of poles and zeros.
5. **Asymptotes:** For large values of s, the asymptotes are straight lines and are symmetric. The angle of the asymptotes is given by

$$\phi_A = \frac{(2q+1)}{n-m} * \pi \tag{6}$$

where q = 0,1,2,...(n-m-1) The point of intersection is given by

$$s_1 = \frac{1}{n-m}\left(\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i\right) \tag{7}$$

# Rules

6 **Multiple roots:** Let $s_0$ be a point on the root locus corresponding to gain $K_0$. By expanding the characteristic equation using Taylor series

$$P(s_0) + K_0Q(s_0) + (s - s_0)[P'(s_0) + K_0Q'(s_0)] + \frac{1}{2}(s - s_0)^2[P''(s_0) + K_0Q''(s_0)] + \quad (8)$$

$$.... + (K - K_0)Q(s_0) + (K - K_0)(s - s_0)Q'(s_0) + ... = 0 \quad (9)$$

So for a double root, the third term of the equation disappears. Thus in the near vicinity of $s_0$ we get the following equation

$$(s - s_0)^2 + 2(K - K_0)Q(s_0)[P''(s_0) + K_0Q''(s_0)]^{-1} = 0 \quad (10)$$

$$arg(s - s_0) = \frac{1}{2}arg(K - K_0) + \alpha/2 + k\pi \quad (11)$$

where

$$\alpha = argQ(s_0) - arg[P''(s_0) + K_0Q''(s_0)] \quad (12)$$

or

$$arg(s - s_0) = \begin{cases} \alpha/2 + k\pi & K < K_0 \\ (\alpha + \pi)/2 + k\pi & K > K_0 \end{cases} \quad k = 1, 2.. \quad (13)$$