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Control System Synthesis - Robust control

PHD CLASS - FALL 2020



Uncertainty and robustness

Where does uncertainty come from?

Modelling uncertainty

Robustness

Small gain theorem

Robust stability

Robust performance

Robust synthesis

$\mathcal{H}_\infty$ -synthesis

$\mathcal{H}_\infty$ -Loopshaping synthesis

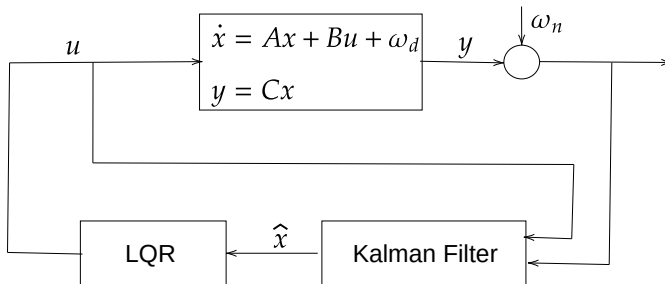
μ -analysis and synthesis

1 Introduction

2 Fundamentals

3 Design techniques

- PID control
- Optimal control and LQG



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3 Design techniques

- PID control
- Optimal control and LQG
- **Robust control and H_∞ synthesis**
- Model Predictive Control
- Adaptive Control
- Data-driven Control

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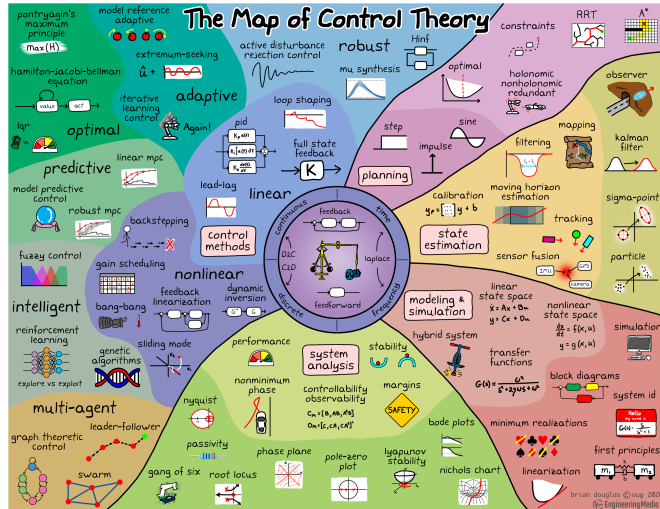
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Content overview

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A model is only an approximation of the reality!

■ Complex dynamics



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- Complex dynamics
- Uncertain inputs
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A model is only an approximation of the reality!

- Complex dynamics
- Uncertain inputs
- Simplified models
- Process variations

The controller is tailored for the model

- Is your controller good enough for the real system? → **analysis**
- Can you take into account the uncertainties during the design? → **synthesis**



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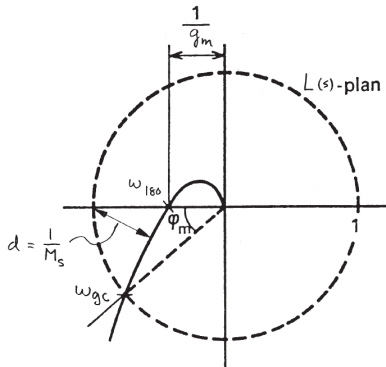


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How to handle uncertainty?

Classical margins requirements

- Gain margin $g_m > 5\text{dB}$
- Phase margin $\varphi_m > 45^\circ$



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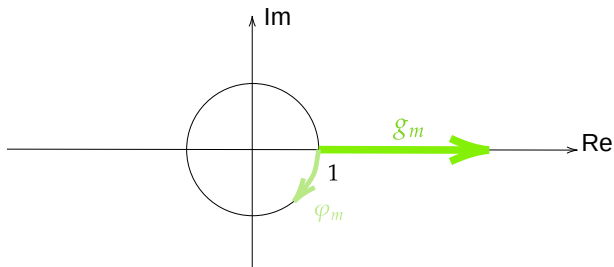
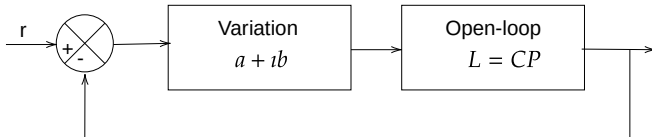
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Limitations of classical margins



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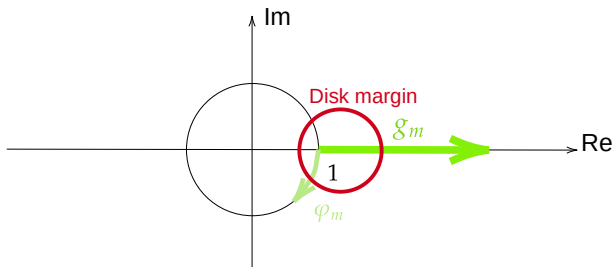
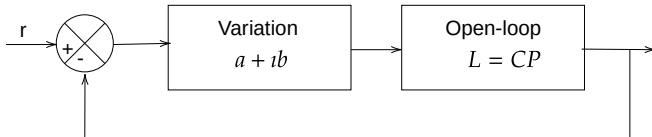
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Less structured representations of uncertainty
→ set of transfer functions around the nominal model



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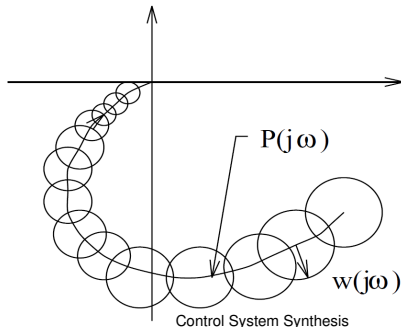
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Less structured representations of uncertainty
→ set of transfer functions around the nominal model

Additive uncertainty

$$P_\Delta(s) = P(s) + W_1(s)\Delta(s)W_2(s), \quad \forall \omega > 0 \quad \bar{\sigma}(\Delta(j\omega)) < 1$$

W_1 and W_2 : spatial and frequency structure of the uncertainty



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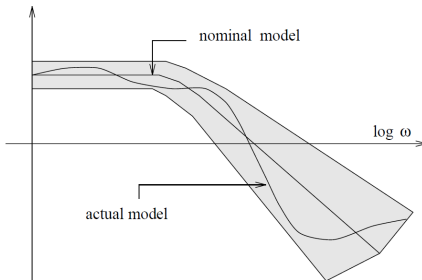


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Less structured representations of uncertainty
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Multiplicative uncertainty

$$P_\Delta(s) = (I + W_1(s)\Delta(s)W_2(s))P(s), \quad \forall \omega > 0 \quad \bar{\sigma}(\Delta(j\omega)) < 1$$



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Definition of robustness

Ability to meet requirements (stability and performances) under disturbance and model uncertainty



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Definition of robustness

Ability to meet requirements (stability and performances) under disturbance and model uncertainty

- Nominal stability: $S = (I + PC)^{-1}$ stable
- Nominal performance: $\bar{\sigma}(S) \leq 1/|W_p|$



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How to analyze/enforce robust stability and performance?

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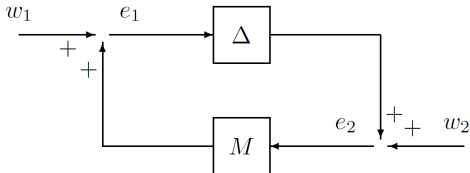
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Small-gain theorem

Suppose $M \in \mathcal{RH}_\infty$ and let $\gamma > 0$. Then this interconnected system is well-posed and internally stable for all $\Delta(s) \in \mathcal{RH}_\infty$ with:

- $\|\Delta\|_\infty \leq \frac{1}{\gamma}$ if and only if $\|M\|_\infty < \gamma$
- $\|\Delta\|_\infty < \frac{1}{\gamma}$ if and only if $\|M\|_\infty \leq \gamma$



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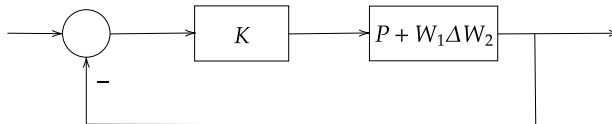
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$$P_\Delta(s) = P(s) + W_1(s)\Delta(s)W_2(s), \quad \forall \omega > 0 \quad \bar{\sigma}(\Delta(j\omega)) < 1$$



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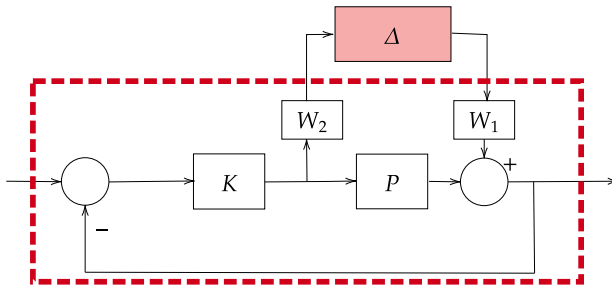
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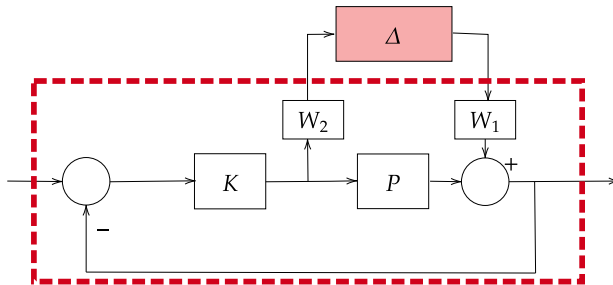
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$$P_\Delta(s) = P(s) + W_1(s)\Delta(s)W_2(s), \quad \forall \omega > 0 \quad \bar{\sigma}(\Delta(j\omega)) < 1$$



The closed-loop is internally stable for $\Delta \in \mathcal{RH}_\infty$ such that $\|\Delta\|_\infty < 1$ if and only if $\|M\|_\infty \leq 1$

$$M = W_2 K (I + KP)^{-1} W_1$$



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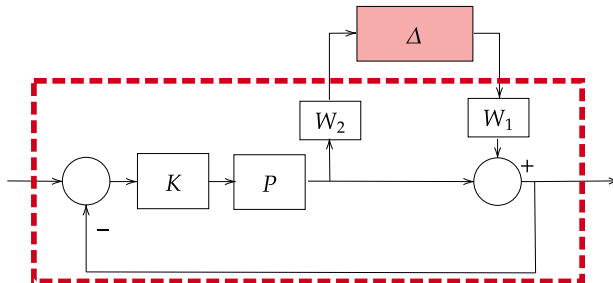
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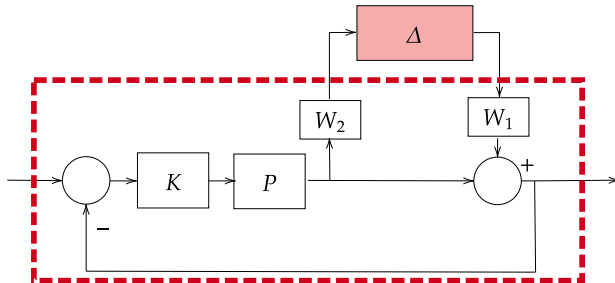
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$W_1 \in \mathcal{RH}_\infty \quad W_2 \in \mathcal{RH}_\infty \quad \Delta \in \mathcal{RH}_\infty \quad \ \Delta\ _\infty < 1$		
Perturbed Model Sets Π	Representative Types of Uncertainty Characterized	Robust Stability Tests
$(I + W_1 \Delta W_2)P$	output (sensor) errors neglected HF dynamics uncertain rhp zeros	$\ W_2 T_o W_1\ _\infty \leq 1$
$P(I + W_1 \Delta W_2)$	input (actuators) errors neglected HF dynamics uncertain rhp zeros	$\ W_2 T_i W_1\ _\infty \leq 1$
$(I + W_1 \Delta W_2)^{-1}P$	LF parameter errors uncertain rhp poles	$\ W_2 S_o W_1\ _\infty \leq 1$
$P(I + W_1 \Delta W_2)^{-1}$	LF parameter errors uncertain rhp poles	$\ W_2 S_i W_1\ _\infty \leq 1$
$P + W_1 \Delta W_2$	additive plant errors neglected HF dynamics uncertain rhp zeros	$\ W_2 K S_o W_1\ _\infty \leq 1$
$P(I + W_1 \Delta W_2 P)^{-1}$	LF parameter errors uncertain rhp poles	$\ W_2 S_o P W_1\ _\infty \leq 1$
$(\tilde{M} + \tilde{\Delta}_M)^{-1}(\tilde{N} + \tilde{\Delta}_N)$ $P = \tilde{M}^{-1} \tilde{N}$ $\Delta = \begin{bmatrix} \tilde{\Delta}_N & \tilde{\Delta}_M \end{bmatrix}$	LF parameter errors neglected HF dynamics uncertain rhp poles & zeros	$\left\ \begin{bmatrix} K \\ I \end{bmatrix} S_o \tilde{M}^{-1} \right\ _\infty \leq 1$
$(N + \Delta_N)(M + \Delta_M)^{-1}$ $P = N M^{-1}$ $\Delta = \begin{bmatrix} \Delta_N \\ \Delta_M \end{bmatrix}$	LF parameter errors neglected HF dynamics uncertain rhp poles & zeros	$\ M^{-1} S_i [K \ I]\ _\infty \leq 1$

Table 8.1: Unstructured robust stability tests (HF: high frequency, LF: low frequency)

Essentials of robust control
Zhou, K., Doyle, J. C. (1998).

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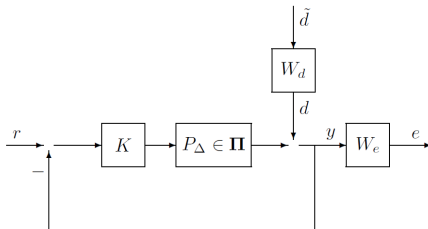
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$$\text{robust stability and } \sup_{\|\tilde{d}\|_2 \leq 1} \|e\|_2 \leq 1$$



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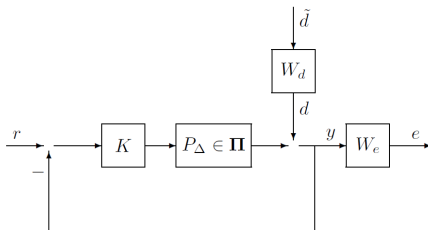
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robust stability and $\|T_{\tilde{d}e}\|_\infty \leq 1, \forall \Delta \in \mathcal{RH}_\infty, \|\Delta\|_\infty < 1$

$$T_{\tilde{d}e} = W_e(I + P_\Delta K)^{-1} W_d \text{ with } P_\Delta \in \Pi$$



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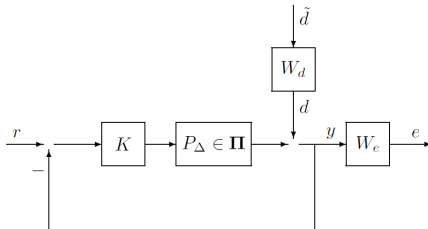
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Sufficient condition in the case of multiplicative uncertainty:

$$\forall \omega, \bar{\sigma}(W_d)\bar{\sigma}(W_e S) + \bar{\sigma}(W_1)\bar{\sigma}(W_2 T) \leq 1$$

$$S = (I + KP)^{-1}, \quad T = I - S$$



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The goal of robust control is to address uncertainty



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The goal of robust control is to address uncertainty

■ Requires to represent uncertainty

■ Margins



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The goal of robust control is to address uncertainty

- Requires to represent uncertainty
 - Margins
 - Model set for the system's transfer function



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The goal of robust control is to address uncertainty

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 - Margins
 - Model set for the system's transfer function
- Small-gain theorem $\rightarrow$ Robust stability test
- Additional sufficient conditions for robust performance



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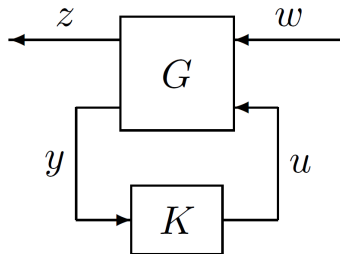
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- **Optimal $\mathcal{H}_\infty$ Control:** Find all admissible controllers $K(s)$ such that $\|T_{zw}\|_\infty$ is minimized.



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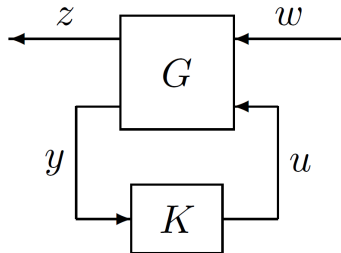
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- **Optimal $\mathcal{H}_\infty$ Control:** Find all admissible controllers $K(s)$ such that $\|T_{zw}\|_\infty$ is minimized.
- **Suboptimal $\mathcal{H}_\infty$ Control:** Given $\gamma > 0$, find all admissible controllers $K(s)$ such that $\|T_{zw}\|_\infty < \gamma$.



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Robust synthesis

$\mathcal{H}_\infty$ -synthesis

Uncertainty and robustness

Where does uncertainty come from?

Modelling uncertainty

Robustness

Small gain theorem

Robust stability

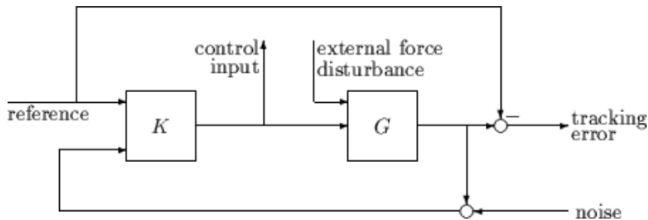
Robust performance

Robust synthesis

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$\mathcal{H}_\infty$ -Loopshaping synthesis

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- Tracking
- Disturbance rejection
- Measurement noise
- Moderate command signal



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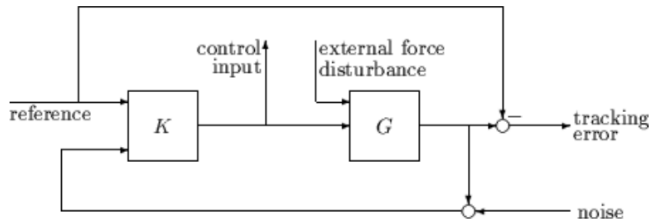
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- Tracking
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$\Rightarrow$ regulated variables = T (external signals)



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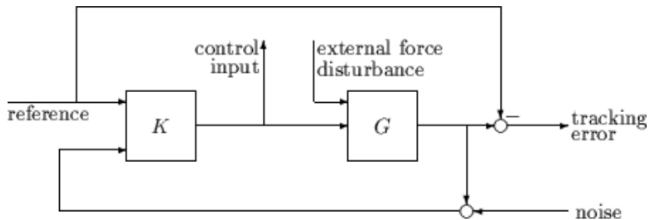
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- Tracking
- Disturbance rejection
- Measurement noise
- Moderate command signal

⇒ regulated variables = T (external signals)
Performance as Generalized Disturbance Rejection

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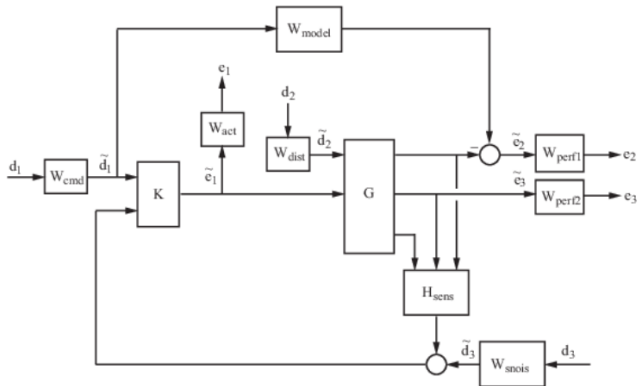
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Generalized and Weighted Performance Block Diagram



Use of weights to handle magnitudes and frequency dependency

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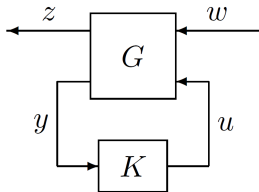
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Resolution

- Matlab Robust control toolbox:
 $K, CL, \gamma = \text{hinfsyn}(P, n_{\text{meas}}, n_{\text{cont}}, \gamma_{\text{max}})$



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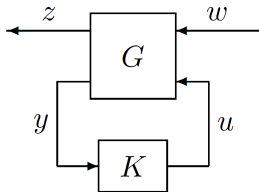
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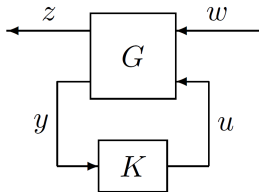
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Resolution

- Matlab Robust control toolbox:
 $K, CL, \gamma = \text{hinfsyn}(P, n_{\text{meas}}, n_{\text{cont}}, \gamma_m)$
 - Riccati
 - LMIs



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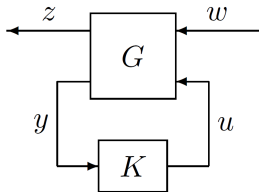
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Resolution

- Matlab Robust control toolbox:
 $K, CL, \gamma = \text{hinfsyn}(P, n_{\text{meas}}, n_{\text{cont}}, \gamma)$
 - Riccati
 - LMIs
- K has the same order than P + centralized structure



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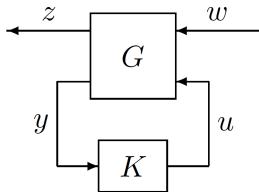
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→ structured version with `hinfstruct`



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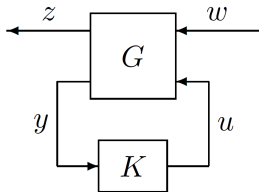
- Riccati

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- K has the same order than P + centralized structure

→ structured version with `hinfstruct`

→ Controller reduction



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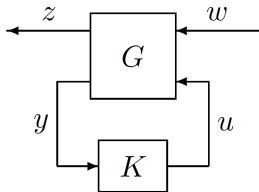
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 $K, CL, \gamma = \text{hinfsyn}(P, n_{\text{meas}}, n_{\text{cont}}, \gamma)$
 - Riccati
 - LMIs
- K has the same order than P + centralized structure
- structured version with `hinfstruct`
- Controller reduction
- Difficulty to tune the weights



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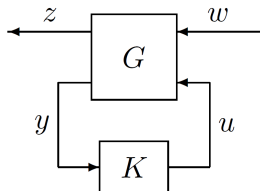
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Resolution

- Matlab Robust control toolbox:
 $K, CL, \gamma = \text{hinfsyn}(P, n_{\text{meas}}, n_{\text{cont}}, \gamma)$
 - Riccati
 - LMIs
- K has the same order than P + centralized structure
- structured version with `hinfstruct`
- Controller reduction
- Difficulty to tune the weights
- Use `systune` : Specify only high-level design requirements (reference tracking, overshoot, disturbance rejection, or open-loop stability margins)

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Further reading

1 Riccati approach

- Doyle, J.C., K. Glover, P. Khargonekar, and B. Francis. *State-space solutions to standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems*. IEEE Transactions on Automatic Control, 1989.

2 LMI approach

- Gahinet, P., and P. Apkarian. *A linear matrix inequality approach to $\mathcal{H}_\infty$ -control*. International Journal of Robust and Nonlinear Control, 1994.

3 Structured $\mathcal{H}_\infty$ -controller design

- P. Apkarian and D. Noll, *Nonsmooth H -infinity Synthesis*, IEEE Transactions on Automatic Control, 2006.

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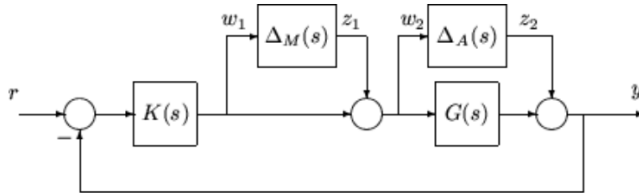
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Robustness in the $\mathcal{H}_\infty$ framework



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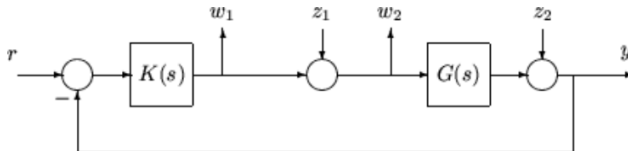
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Robustness in the $\mathcal{H}_\infty$ framework



- Keep $T_{z_1 w_1}$ and $T_{z_2 w_2}$ small
- Robustness objectives = additional "disturbance to error" transfer functions to be kept small

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Glover-McFarlane Method Step 1: Loop-shaping

Considering the nominal plant P , shape the desired open-loop through a precompensator W_1 and a postcompensator W_2

$$P_s = W_2 P W_1$$

Typical good open-loop design

- Large gain in the low frequency region (disturbance rejection)

$$\underline{\sigma}(PK) \gg 1$$

- Small gain in the high frequency region:

$$\overline{\sigma}(PK) \ll 1$$

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Glover-McFarlane Method Step 2: Robust stabilization

- Normalize coprime factorization $P_s = M^{-1}N$



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Glover-McFarlane Method Step 2: Robust stabilization

- Normalize coprime factorization $P_s = M^{-1}N$
- Uncertainty $P_\Delta = (M + \Delta_M)^{-1}(N + \Delta_N)$ with $\Delta_M, \Delta_N \in \mathcal{RH}_\infty$ and $\|[\Delta_M, \Delta_N]\|_\infty < \varepsilon$



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- **Small-gain theorem:** if K is stabilizing, the closed-loop is robustly stable iff

$$\left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + P_s K)^{-1} M^{-1} \right\|_\infty \leq \frac{1}{\varepsilon}$$



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→ you want to have ε as big as possible. If ε_{max} is not big enough, redesign W_1 and W_2

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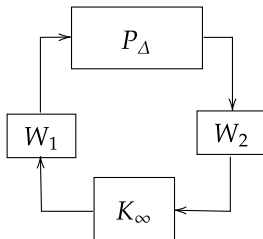
Glover-McFarlane Method

Step 3: Combine loop-shaping and robust stabilization

- Synthesize a controller K_∞ satisfying

$$\left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + P_s K)^{-1} M^{-1} \right\|_\infty \leq \frac{1}{\varepsilon}$$

with $\varepsilon \leq \varepsilon_{max}$



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Glover-McFarlane Method

Step 3: Combine loop-shaping and robust stabilization

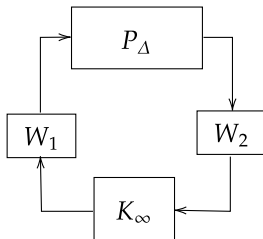
- Synthesize a controller K_∞ satisfying

$$\left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + P_s K)^{-1} M^{-1} \right\|_\infty \leq \frac{1}{\varepsilon}$$

with $\varepsilon \leq \varepsilon_{max}$

- The final controller is given by

$$K = W_1 K_\infty W_2$$



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Motivations:

- Loopshaping is done without handling closed-loop stability requirements.
- Robust stabilization is done without frequency weighting.

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Motivations:

- Loopshaping is done without handling closed-loop stability requirements.
- Robust stabilization is done without frequency weighting.
- ε is a measure of both closed-loop robust stability and the success of the design in meeting the loop-shaping specifications.

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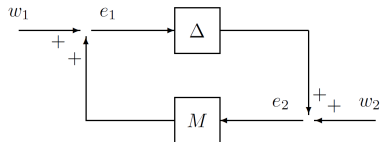
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Uncertainty may be modeled in two ways:

- as external inputs



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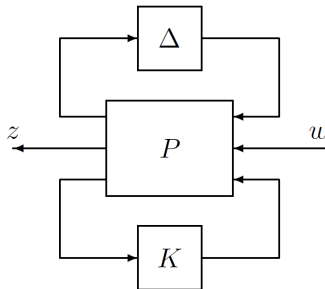
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Uncertainty may be modeled in two ways:

- as external inputs
- as perturbations to the nominal model



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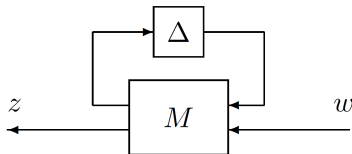
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Analysis framework



Δ is a structured block

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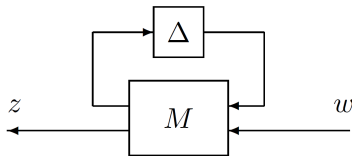
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Analysis framework



Δ is a structured block

μ -analysis

Let $\beta > 0$. Then this interconnected system is well-posed and internally stable for all $\Delta(s) \in \mathcal{M}(\Delta)$ with $\|\Delta\|_\infty \leq 1/\beta$, and $\|\mathcal{F}(N, \Delta)\|_\infty \leq \beta$, if and only if

$$\sup_{\omega \in \mathbb{R}} \mu_\Delta(M(j\omega)) \leq \beta$$



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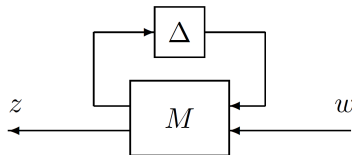
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Analysis framework



Δ is a structured block

- $\mathcal{F}(N, \Delta)$ is the transfer from w to z
- μ_Δ is defined as

$$\mu_\Delta(M) = \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in \mathcal{M}(\Delta), \det(I - M\Delta) = 0\}}$$

unless no Δ makes $I - M\Delta$ singular, in which case $\mu_\Delta(M) = 0$.

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→ Iterative process called D-K iteration

- 1 Uses $\mathcal{H}_\infty$ -synthesis to find a controller that minimizes the closed-loop gain of the nominal system.



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→ Iterative process called D-K iteration

- 1 Uses $\mathcal{H}_\infty$ -synthesis to find a controller that minimizes the closed-loop gain of the nominal system.
- 2 **D-step:** Robustness analysis to estimate μ_Δ and a new scaling D .



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μ -synthesis

→ Iterative process called D-K iteration

- 1 Uses $\mathcal{H}_\infty$ -synthesis to find a controller that minimizes the closed-loop gain of the nominal system.
- 2 **D-step:** Robustness analysis to estimate μ_Δ and a new scaling D .
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Robust synthesis

μ -analysis and synthesis

Uncertainty and robustness

Where does uncertainty come from?

Modelling uncertainty

Robustness

Small gain theorem

Robust stability

Robust performance

Robust synthesis

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→ Iterative process called D-K iteration

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Extremely computationally demanding!



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