Control System Synthesis - PhD Class

Exercise session 2

October 8, 2020

1 Inverted pendulum on a cart

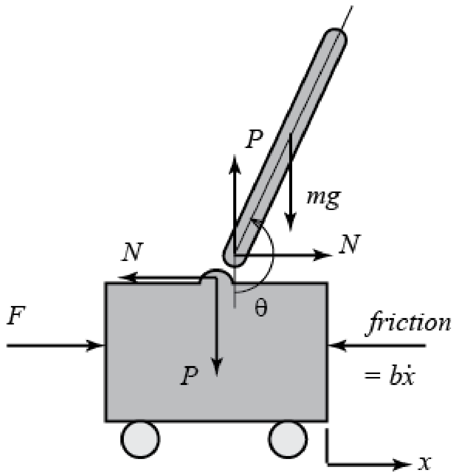

Figure 1: Inverted pendulum.

The equations of motion are :

$$\begin{aligned}(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \\ (J+ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta\end{aligned} \tag{1}$$

where:

- $M = 0.5$kg is the mass of the cart
- $m = 0.2$kg is the mass of the pendulum
- $b = 0.1$N/m/sec is the coefficient of friction for the cart
- $l = 0.3$m is the length to pendulum center of mass
- $J = 0.006 kg.m^2$ is the mass moment of inertia of the pendulum
- F is the force applied to the cart
- x is the cart position
- θ is the pendulum angle from vertical

Objectives

Along this exercise, the objective is to stabilize the inverted pendulum in the upward position. The design requirements are the following:

- Settling time for x and θ of less than 5 seconds
- Pendulum angle θ never more than 4 degrees (0.07 radians) from the vertical

For this exercise, there is a matlab skeleton available on the course page.

1.1 About the system

1. Assuming that $\theta = \pi + \phi$ with ϕ is a small deviation from the equilibrium, write linear equations of motion using small angle approximation:

$$\cos\theta = \cos(\pi + \phi) \approx -1,$$

$$\sin\theta = \sin(\pi + \phi) \approx -\phi,$$
$$\dot{\theta}^2 = \dot{\phi}^2 \approx 0.$$

2. Write the linearized equations of motion in a state-space form. What is the state vector x? the output vector y? the input u?

3. Is this system controllable? (use Matlab) and what does that imply?

In the next two parts, we will design full state feedback controllers through pole placement and LQR.

1.2 Pole placement

Given $\dot{x} = Ax + Bu$ and a vector p of desired closed-loop pole locations, the matlab command

$$K = place(A, B, p)$$

computes a gain matrix K such that the state feedback $u = -Kx$ places the closed-loop poles at the locations p. In other words, the eigenvalues of $A–BK$ match the entries of p.

1. What are the poles of this system? (use Matlab)

2. Run the given Matlab script withthe given p. How should you modify the desired closed-loop poles to obtain a better behaviour?

3. Propose a new choice for p based on the design requirements and simulate the results.

1.3 LQR

1. Run the given Matlab script with the given weights Q and R. How should you modify the desired closed-loop poles to obtain a better behaviour?

2. Propose a new choice for p based on the design requirements and simulate the results.

1.4 Bonus: Observer-based control

If you want to go further, add a state-estimator to obtain a LQG controller.

2 $\mathcal{H}-\infty$ loopshaping (Glover-McFarlane)

Consider the following plant:

$$P(s) = \frac{1}{6s2 + 0.16s + 16}.$$

2.1 Design Objectives and loopshaping

The design objectives for the closed-loop are the following:

- Noise attenuation beyond 20 rad/sec
- Tracking:
  - Zero steady-state error (integral action)
  - Bandwidth of at least 0.5 rad/s
- Gain crossover frequencies no larger than 7 rad/s

First, we want to translate these requirements into a desired shape L_d for the open-loop gain and seek a compensator W that enforces this shape:

$$L_d \approx PW$$

2.2 Enforcing stability and robustness

You can use the Matlab command *ncfsyn* to enforce stability and adequate stability margins without significantly altering the loop shape. Take a look at the documentation.

Use the initial design W of the compensator to obtain a controller.

- How can you tell if your compensator is good enough? Redesign it if necessary.
- To analyse the results, take a look at the margins, the closed-loop impulse response and the Bode plots of the sensitivity and complementary sensitivity functions.