Session 2

Dissipativity and Integral Quadratic Constraints

Reading assignment

You don't need to read everything from these papers, but check the main results and some examples. Jan C. Willems was the leading figure of systems and control in the Netherlands for several decades. The other two papers are from our department.

- Jan C. Willems, Arch. Rational Mech. and Analysis, 45:5 (1972).
- A. Megretski and A. Rantzer, IEEE TAC 42:6 (1997).
- Chung-Yao Kao and Bo Lincoln, Automatica 40 (2004).

Exercise 2.1 Let $\Pi = \begin{bmatrix} X & Y \\ Y^T & -X \end{bmatrix}$, where $X = X^T \succeq 0$ and $Y = -Y^T$. Verify the IQC defined by Π when $w = \Delta(v)$ means multiplication by a time-varying scalar: $w(t) = \delta(t)v(t)$ where $\delta \in [-1, 1]$.

Exercise 2.2
Consider feedback control with plant $P(s) = \frac{1}{s(s+1)}$ and controller $K(s) = \frac{30s+30}{s+9}$. You know from earlier courses how to compute the delay margin for a closed loop system. You should also know how to prove stability of the system in presence of input saturation using the circle criterion. But what if you have both delay and saturation? Or if the delay is time-varying? This is what this excercise is about:

a. Determine the matrices A, B and C from $P(s)$ and $K(s)$ to write the closed loop system with input saturation and delay on the form

$$\dot{x}(t) = Ax(t) + B\,\mathrm{sat}(Cx(t-\tau)) \qquad (\star)$$

b. Give a frequency domain inequality that is sufficient to guarantee stability of the nonlinear system $(\star)$ for $\tau \in [0, T]$.
c. Give a linear matrix inequality that is sufficient to guarantee stability of the system for $\tau \in [0, T]$. What is the largest possible value of T for which you can prove stability?
d. Construct a storage functions and supply rates for the LTI system, the saturation and the uncertain delay.
e. Consider the same system without saturation but with time-varying delays using the work of Kao and Lincoln. What is the maximal value of T for which you can prove that $\dot{x}(t) = Ax(t) + BCx(t - \tau(t))$ is stable when $\tau(t) \in [0, T]$?
f. Answer the same question with both saturation and time-varying delay using the Matlab toolbox IQCbeta.

Exercise 2.3 Formulate a discrete time version of Theorem 1 in Megretski/Rantzer 1997. You don't need to prove it.