Boiler Modeling

Goal: To present a major industrial modeling effort (pre Modelica). Practice balance equations. To illustrate that it takes time to obtain good simple models.

Rodney Bell: Nature does not willingly part with its secrets!

1. Introduction
2. Global Balance Equations
3. Steam Distribution
4. The Model
5. Simulation
6. Experiments
7. Conclusions

Introduction

- Long term research project
- Strong support from Sydkraft AB on many levels
- Many directions
  - Level control
  - Physical modeling and control
  - Modeling tools
- Strong international collaboration (Rodney Bell)
- Many PhDs and masters projects

Kalle Eklund	Physical Modeling
Sture Lindahl	Dymola
Hilding Elmqvist	Omola/Omsim/Modelica
Jonas and Hubertus	K2 library

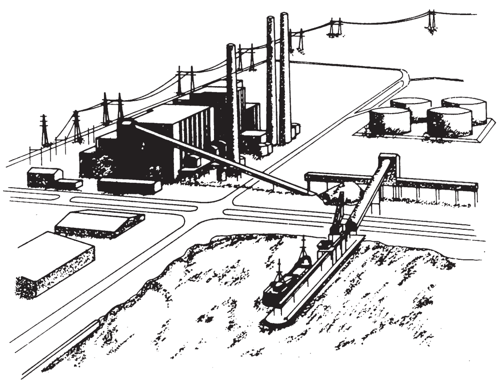
Boiler Modeling

- Complex system
  - Many components
  - Large dimensions
  - Complicated physics
  - Two phase flow
  - Interesting behavior (Shrink and swell)
  - Steam tables
- Steam gives a strong coupling
- Subsystem decomposition
- Global balance equations for each subsystem
- Differential algebraic equations
- Ideal for Modelica

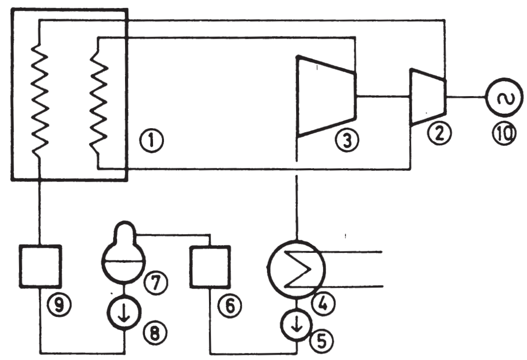
Why is it of Interest

- Has changed over the years
- 30% of emergency shut downs of French nuclear reactors attributable to level control problems in steam generators
- EDF Benchmark at IFAC Beijing
- Model based control
- Modeling groups
  - Stuttgardt Quazza, Welfonder
  - ETH Profos
  - Milan Maffezoni
  - Lund
  - Philadelphia Electric Harry Kwatny
  - Australia Rodney Bell et al

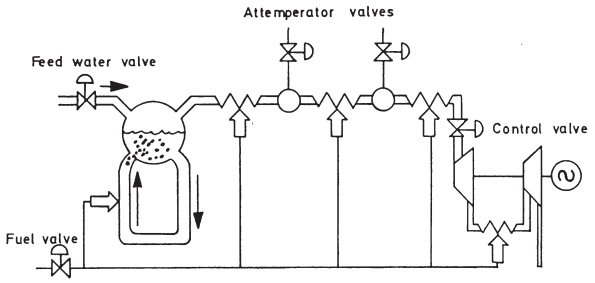
Öresundsverket



The P16-G16 Unit



A Schematic Diagram



Schematic diagram of the boiler-turbine unit.

The Modeling Process

- Physics
- Simulation
- Experimental data
- Importance of P15-G16 data
- Try many assumptions
- Simplify
- Guidelines
  - Base as much as possible on physics
  - Keep it simple and transparent
  - Explain the experimental data
  - Match the shapes

Modeling Features

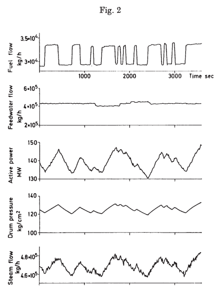
- Should captures many properties of importance for control
- Should work over a wide operating range
- Global mass, energy and momentum balances
- Steam tables
- Parameters from construction data
- Models drum level (shrink and swell) very well
- A new phenomena "rapid swell" has been found
- Why low order?

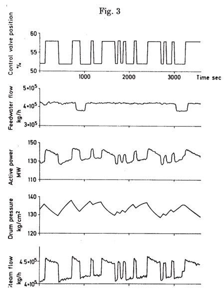
Where is the Energy Stored?

Boiler	*Metal*	*Water*	*Steam*	*Total*
P16-G16 80 MW	641	739	64	1444
P16-G16 160 MW	320	333	37	690
Eraring 660 MW	1174	303	60	1537
Eraring 330 MW	587	137	35	759

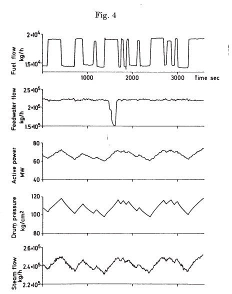
Unit: $T = \frac{StoredEnergy}{Power}$ [J/W=s]

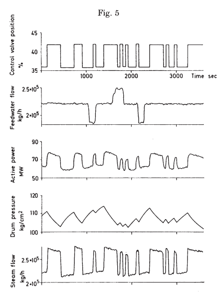
Experimental Data 1



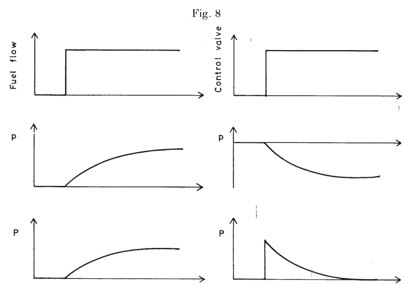


Experimental Data 1





Qualitative Behavior 1



The low order mystery!

System identification indicates that models of order 3-5 is sufficient

The First Simple Model Åström Eklund 1972

Global energy balance
Fit to data

$$\frac{dp}{dt} = -\alpha_1(u_2 p^{5/8} - \alpha_5) + \alpha_2 u_1 - \alpha_3 u_3$$
$$P = \alpha_4(u_2 p^{5/8} - \alpha_5$$

- p drum pressure
- P output power
- u_1 fuel flow
- u_2 steam valve opening
- u_3 feedwater flow

The Never Ending Story

1. Physical modeling
2. Experiments
3. Identification (Good indicator of dependencies and complexity)
4. Look at data
5. Add terms for better fit
6. Try to explain physically
7. Simplify
8. Back to 1

Model complexity shrinks and grows. Not a good thing to do if you want to publish many papers.

Lecture 9 - Boiler Modeling

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The Steam Generator

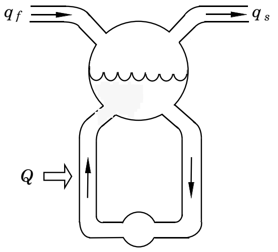
How to cut the system?

Inputs:

- Fuel flow
- Feedwater flow
- Feedwater temperature
- Steam flow

Outputs:

- Drum pressure
- Drum level
- Steam quality



Sensible physical approximations

Global Balance Equations

Mass balance

$$\frac{d}{dt}\left[\varrho_s V_{st} + \varrho_w V_{wt}\right] = q_f - q_s$$

Energy balance

$$\frac{d}{dt}\left[\varrho_s u_s V_{st} + \varrho_w u_w V_{wt} + m_t C_p t_m\right] = Q + q_f h_f - q_s h_s$$

Since $u = h - p/\varrho$ we get

$$\frac{d}{dt}\left[\varrho_s h_s V_{st} + \varrho_w h_w V_{wt} - pV_t + m_t C_p t_m\right] = Q + q_f h_f - q_s h_s$$

Choose pressure p and V_{wt} as state variables

Second Order Model

$$\frac{d}{dt}\big(\varrho_s V_{st} + \varrho_w V_{wt}\big) = q_f - q_s$$
$$\frac{d}{dt}\big(\varrho_s h_s V_{st} + \varrho_w h_w V_{wt} - pV_t + m_t C_p t_m\big) = Q + q_f h_f - q_s h_s$$

$$e_{11}\frac{dV_{wt}}{dt} + e_{12}\frac{dp}{dt} = q_f - q_s$$
$$e_{21}\frac{dV_{wt}}{dt} + e_{22}\frac{dp}{dt} = Q + q_f h_f - q_s h_s$$
$$e_{11} = \varrho_w - \varrho_s$$
$$e_{12} = V_{st}\frac{\partial \varrho_s}{\partial p} + V_{wt}\frac{\partial \varrho_w}{\partial p}$$
$$e_{21} = \varrho_w h_w - \varrho_s h_s$$
$$e_{22} = V_{st}\Big(h_s\frac{\partial \varrho_s}{\partial p} + \varrho_s\frac{\partial h_s}{\partial p}\Big) + V_{wt}\Big(h_w\frac{\partial \varrho_w}{\partial p} + \varrho_w\frac{\partial h_w}{\partial p} - V_t + m_t C_p\frac{\partial t_s}{\partial p}$$

First Order Model

Total mass and energy balances

$$\frac{d}{dt}\left[\varrho_s V_{st} + \varrho_w V_{wt}\right] = q_f - q_s,$$
$$\frac{d}{dt}\left[\varrho_s u_s V_{st} + \varrho_w u_w V_{wt} + m_t C_p t_m\right] = Q + q_f h_f - q_s h_s.$$

Eliminate the derivative of V_{wt}

$$h_c\frac{d}{dt}\left(\varrho_s V_{st}\right) + \varrho_s V_{st}\frac{dh_s}{dt} + \varrho_w V_{wt}\frac{dh_w}{dt} - V_t\frac{dp}{dt} + m_t C_p\frac{dt_s}{dt}$$
$$= Q - q_f\left(h_w - h_f\right) - q_s h_c,$$

Hence

$$e_1\frac{dp}{dt} = Q - q_f(h_w - h_f) - q_s h_c,$$
$$e_1 = h_c V_{st}\frac{\partial \varrho_s}{\partial p} + \varrho_s V_{st}\frac{\partial h_s}{\partial p} + \varrho_w V_{wt}\frac{\partial h_w}{\partial p} + m_t C_p\frac{\partial t_s}{\partial p} - V_t.$$

Which Terms are Important?

First order model

$$e_1\frac{dp}{dt} = Q - q_f(h_w - h_f) - q_s h_c,$$
$$e_1 = h_c V_{st}\frac{\partial \varrho_s}{\partial p} + \varrho_s V_{st}\frac{\partial h_s}{\partial p} + \varrho_w V_{wt}\frac{\partial h_w}{\partial p} + m_t C_p\frac{\partial t_s}{\partial p} - V_t.$$

Boiler	$h_c V_{st}\frac{\partial \varrho_s}{\partial p}$	$\varrho_s V_{st}\frac{\partial h_s}{\partial p}$	$\varrho_w V_{wt}\frac{\partial h_w}{\partial p}$	$m_t C_p\frac{\partial t_s}{\partial p}$	V_t
P16 80 MW	360	-40	2080	1410	85
P16 160 MW	420	-40	1870	1410	85
E 330 MW	700	-270	2240	4620	169
E 660 MW	810	-270	2020	4620	169

Condensation Flow Rate

First order model

$$e_1\frac{dp}{dt} = Q - q_f(h_w - h_f) - q_s h_c,$$
$$e_1 = h_c V_{st}\frac{\partial \varrho_s}{\partial p} + \varrho_s V_{st}\frac{\partial h_s}{\partial p} + \varrho_w V_{wt}\frac{\partial h_w}{\partial p} + m_t C_p\frac{\partial t_s}{\partial p} - V_t$$

The model can be written as follows

$$Q + h_c q_{ct} = h_c q_s$$

where q_{ct} is the total condensation flow rate, hence

$$q_{ct} = \frac{h_w - h_f}{h_c}q_f + \frac{1}{h_c}\Big(\varrho_s V_{st}\frac{dh_s}{dt} + \varrho_w V_{wt}\frac{dh_w}{dt} - V_t\frac{dp}{dt} + m_t C_p\frac{dt_s}{dt}\Big).$$

Lecture 9 - Boiler Modeling

Steam Distribution

The first order model which is a global energy balance does not describe the distribution of steam and water in the system. This is important do deal with several problems in the boiler

- Level control the shrink and swell effect
- Drives the circulation flow in natural circulation boilers
- Two phase flow
- The classic instabilities
- The nuclear reactor experience
- How detailed models are required
- Do we need to model steam both in risers and drum?

Water in a Heated Tube

Mass and energy balances

$$A\frac{\partial \varrho}{\partial t} + \frac{\partial q}{\partial z} = 0$$

$$\frac{\partial \varrho h}{\partial t} + \frac{1}{A}\frac{\partial qh}{\partial z} = \frac{Q}{V}$$

$$h = \alpha_m h_s + (1-\alpha_m)h_w = h_w + \alpha_m(h_s - h_w) = h_w + \alpha_m h_c$$

Steady state solution

$$\frac{\partial q}{\partial z} = 0 \qquad q = const$$

$$\frac{\partial qh}{\partial z} = qh_c\frac{\partial \alpha_m}{\partial z} = \frac{QA}{V} \qquad \alpha_m = \frac{QA}{qh_cV}z$$

Assume this shape dynamically

Mass and Volume Fractions

Basic relation

$$\alpha_v = f(\alpha_m) = \frac{\varrho_w \alpha_m}{\varrho_s + (\varrho_w - \varrho_s)\alpha_m}$$

With linear distribution we get

$$\alpha_m(\xi) = \alpha_r \xi \quad 0 \leq \xi \leq 1$$

Average over riser tube

$$\bar{\alpha}_v = \int_0^1 \alpha_v(\xi) = \frac{1}{\alpha_r}\int_0^{\alpha_r} f(\xi)d\xi$$

$$= \frac{\varrho_w}{\varrho_w - \varrho_s}\left(1 - \frac{\varrho_s}{(\varrho_w - \varrho_s)\alpha_r}\ln\left(1 + \frac{\varrho_w - \varrho_s}{\varrho_s}\alpha_r\right)\right).$$

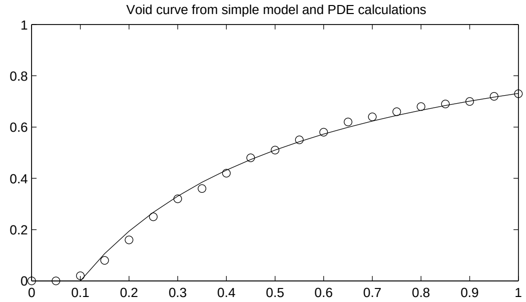
Remarks

- The details of two phase flows are much more complicated than our simple model indicates
- Steam and water flows at different rates and there is slip between them
- We have also made a strong assumption that the shape of the dynamics void profile can be approximated with the steady state profile
- We have also assumed that boiling starts immediately (this is easy to fix)
- Even so the model gives good fit for our purposes

Void Distribution

Comparison with elaborate PDE code based on detailed modeling of two phase flow, the Polka code from Barsebæck.

Void curve from simple model and PDE calculations



Model for Risers

Assume a given shape of the void distribution and integrate!
Mass balance for risers

$$\frac{d}{dt}\left(\varrho_s \bar{\alpha}_v V_r + \varrho_w(1-\bar{\alpha}_v)V_r\right) = q_{dc} - q_r,$$

Energy balance for risers

$$\frac{d}{dt}\left(\varrho_s h_s \bar{\alpha}_v V_r + \varrho_w h_w (1-\bar{\alpha}_v)V_r - pV_r + m_r C_p t_s\right)$$
$$= Q + q_{dc}h_w - (\alpha_r h_c + h_w)q_r.$$

Circulation Flow

Momentum balance for riser downcomer

$$(L_r + L_{dc})\frac{dq_{dc}}{dt} = (\varrho_w - \varrho_s)\bar{\alpha}_v V_r g - \frac{k}{2}\frac{q_{dc}^2}{\varrho_w A_{dc}},$$

Time constant

$$T = \frac{(L_r + L_{dc})A_{dc}\varrho_w}{kq_{dc}}.$$

is about a second. Neglect fast dynamics use static model

$$\frac{1}{2}kq_{dc}^2 = \varrho_w A_{dc}(\varrho_w - \varrho_s)g\bar{\alpha}_v V_r.$$

Distribution of Steam in Drum

Let V_{sd} and V_{wd} be the volume of steam and water under the liquid level and let the steam flow rate through the liquid surface in the drum be q_{sd}.
Mass balance for steam under drum level

$$\frac{d}{dt}\left(\varrho_s V_{sd}\right) = \alpha_r q_r - q_{sd} - q_{cd},$$

Condensation flow

$$q_{cd} = \frac{h_w - h_f}{h_c}q_f + \frac{1}{h_c}\Big(\varrho_s V_{sd}\frac{dh_s}{dt} + \varrho_w V_{wd}\frac{dh_w}{dt}$$
$$- (V_{sd} + V_{wd})\frac{dp}{dt} + m_d C_p \frac{dt_s}{dt}\Big).$$

Steam flow out of the drum

$$q_{sd} = \frac{\varrho_s}{T_d}(V_{sd} - V_{sd}^0) + \alpha_r q_{dc} + \alpha_r \beta(q_{dc} - q_r).$$

Drum Level

Volume of water in the drum

$$V_{wd} = V_{wt} - V_{dc} - (1-\bar{\alpha}_v)V_r.$$

Drum has a complicated geometry, linearize

$$\ell = \frac{V_{wd} + V_{sd}}{A_d} = \ell_w + \ell_s.$$

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The Complete Model

The model is given by mass and energy balances for

- The complete system
- The risers
- The drum

Choice of state variables

- Drum pressure p
- Total water volume V_{wt}
- Steam quality in risers α_m
- Volume of steam in drum V_{sd}

Simplification of Riser Dynamics

Mass and energy balance for risers

$$\frac{d}{dt}\left(\varrho_s \bar{\alpha}_v V_r + \varrho_w (1-\bar{\alpha}_v) V_r\right) = q_{dc} - q_r,$$

$$\frac{d}{dt}\left(\varrho_s h_s \bar{\alpha}_v V_r + \varrho_w h_w (1-\bar{\alpha}_v) V_r - pV_r + m_r C_p t_s\right) = Q + q_{dc} h_w - (\alpha_r h_c + h_w) q_r.$$

Eliminate mass flow rate out of riser tubes q_r!

$$h_c(1-\alpha_r)\frac{d}{dt}\left(\varrho_s \bar{\alpha}_v V_r\right) + \varrho_w (1-\bar{\alpha}_v) V_r \frac{dh_w}{dt} - \alpha_r h_c \frac{d}{dt}\left(\varrho_w (1-\bar{\alpha}_v) V_r\right) + \varrho_s \bar{\alpha}_v V_r \frac{dh_s}{dt} - V_r \frac{dp}{dt} + m_r C_p \frac{dt_s}{dt} = Q - \alpha_r h_c q_{dc}.$$

$$q_r = q_{dc} - V_r \frac{\partial}{\partial p}\left((1-\bar{\alpha}_v)\varrho_w + \bar{\alpha}_v \varrho_s\right)\frac{dp}{dt} + V_r(\varrho_w - \varrho_s)\frac{\partial \bar{\alpha}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt}.$$

The Model

The model can be represented as

$$e_{11}\frac{dV_{wt}}{dt} + e_{12}\frac{dp}{dt} = q_f - q_s$$

$$e_{21}\frac{dV_{wt}}{dt} + e_{22}\frac{dp}{dt} = Q + q_f h_f - q_s h_s$$

$$e_{32}\frac{dp}{dt} + e_{33}\frac{d\alpha_r}{dt} = Q - \alpha_r h_c q_{dc}$$

$$e_{42}\frac{dp}{dt} + e_{43}\frac{d\alpha_r}{dt} + e_{44}\frac{dV_{sd}}{dt} = \frac{\varrho_s}{T_d}(V_{sd}^0 - V_{sd}) + \frac{h_f - h_w}{h_c} q_f,$$

Notice nested triangular structure $(((V_{wt}, p), \alpha_r), V_{sd})$!

Coefficients

$$e_{11} = \varrho_w - \varrho_s$$

$$e_{12} = V_{wt}\frac{\partial \varrho_w}{\partial p} + V_{st}\frac{\partial \varrho_s}{\partial p}$$

$$e_{21} = \varrho_w h_w - \varrho_s h_s$$

$$e_{22} = V_{wt}\left(h_w \frac{\partial \varrho_w}{\partial p} + \varrho_w \frac{\partial h_w}{\partial p}\right) + V_{st}\left(h_s \frac{\partial \varrho_s}{\partial p} + \varrho_s \frac{\partial h_s}{\partial p}\right) - V_t + m_t C_p \frac{\partial t_s}{\partial p}$$

$$e_{32} = \left(\varrho_w \frac{\partial h_w}{\partial p} - \alpha_r h_c \frac{\partial \varrho_w}{\partial p}\right)(1-\bar{\alpha}_v)V_r + \left((1-\alpha_r)h_c \frac{\partial \varrho_s}{\partial p} + \varrho_s \frac{\partial h_s}{\partial p}\right)\bar{\alpha}_v V + \left(\varrho_s + (\varrho_w - \varrho_s)\alpha_r\right)h_c V_r \frac{\partial \bar{\alpha}_v}{\partial p} - V_r + m_r C_p \frac{\partial t_s}{\partial p}$$

$$e_{33} = \left((1-\alpha_r)\varrho_s + \alpha_r \varrho_w\right)h_c V_r \frac{\partial \bar{\alpha}_v}{\partial \alpha_r}$$

Coefficients continued

$$e_{42} = V_{sd}\frac{\partial \varrho_s}{\partial p} + \frac{1}{h_c}\left(\varrho_s V_{sd}\frac{\partial h_s}{\partial p} + \varrho_w V_{wd}\frac{\partial h_w}{\partial p} - V_{sd} - V_{wd} + m_d C_p \frac{\partial t_s}{\partial p}\right) + \alpha_r(1+\beta)V_r\left(\bar{\alpha}_v \frac{\partial \varrho_s}{\partial p} + (1-\bar{\alpha}_v)\frac{\partial \varrho_w}{\partial p} + (\varrho_s - \varrho_w)\frac{\partial \bar{\alpha}_v}{\partial p}\right)$$

$$e_{43} = \alpha_r(1+\beta)(\rho_s - \rho_w)V_r \frac{\partial \bar{\alpha}_v}{\partial \alpha_r}$$

$$e_{44} = \varrho_s.$$

Steam Tables

Properties of steam and water in saturated state

```
hs = a01+(a11+a21*(p-10))*(p-10);
dhsdp = a11+2*a21*(p-10);
rs = a02+(a12+a22*(p-10))*(p-10);
drsdp = a12+2*a22*(p-10);
hw = a03+(a13+a23*(p-10))*(p-10);
dhwdp = a13+2*a23*(p-10);
rw = a04+(a14+a24*(p-10))*(p-10);
drwdp = a14+2*a24*(p-10);
ts = a05+(a15+a25*(p-10))*(p-10);
dtsdp = a15+2*a25*(p-10);
```

Properties of water in subcritical state hf = (Cfw*tf+p*1e3/rw)*1e3;

Parameters

- drum volume V_d,
- riser volume V_r,
- downcomer volume V_{dc},
- drum area A_d at normal operating level,
- total metal mass m_t,
- total riser mass m_r,
- friction coefficient in downcomer-riser loop k,
- residence time T_d of steam in drum,
- parameter β in the empirical equation for riser flow

Equilibria

$$q_f = q_s$$
$$Q = q_s h_s - q_f h_f$$
$$Q = q_{dc} \alpha_r h_c$$
$$V_{sd} = V_{sd}^0 - \frac{T_d (h_w - h_f)}{\varrho_s h_c} q_f,$$

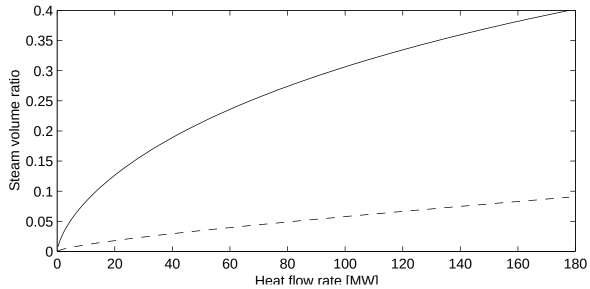
where q_{dc} is given by

$$q_{dc} = \sqrt{\frac{2 \varrho_w A_{dc} (\varrho_w - \varrho_s) g \bar{\alpha}_v V_r}{k}}.$$

Specify q_s and p solve nonlinear equation

$$Q = \alpha_r h_c \sqrt{\frac{2 \varrho_w A_{dc} (\varrho_w - \varrho_s) g \bar{\alpha}_v V_r}{k}}$$
$$\bar{\alpha}_v = \frac{\varrho_w}{\varrho_w - \varrho_s} \Big(1 - \frac{\varrho_s}{(\varrho_w - \varrho_s)\alpha_r} \ln\big(1 + \frac{\varrho_w - \varrho_s}{\varrho_s} \alpha_r\big)\Big).$$

Void Factor



Linearization

The linearized system has

- Two poles at $s = 0$. Physical explanation!
- One pole at $-h_c q_{dc} / e_{33}$
- One pole at $-1/T_d$

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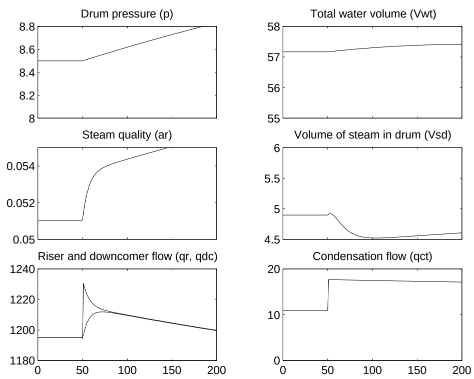
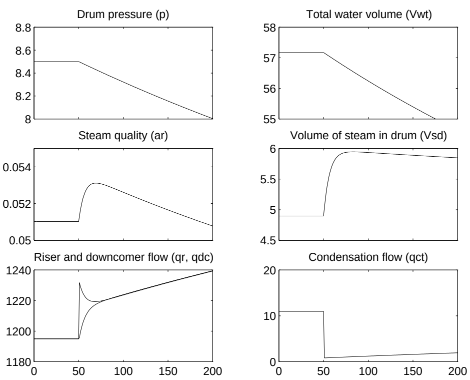
Simulation

- Simulink
- A state model was derive manually
- A lot of tedious error prone manipulations because Simulink cannot deal with constraint models
- Much effort to try different models
- Occasional trouble with integration routine
- Slow!!!
- Workable
- Highly desirable to use Modelica
- A possible project
- How to deal with steam tables?

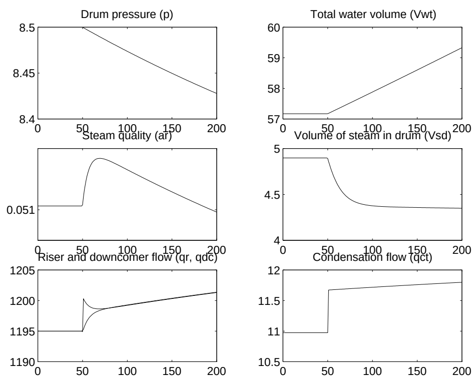
Step in Fuel Flow Rate



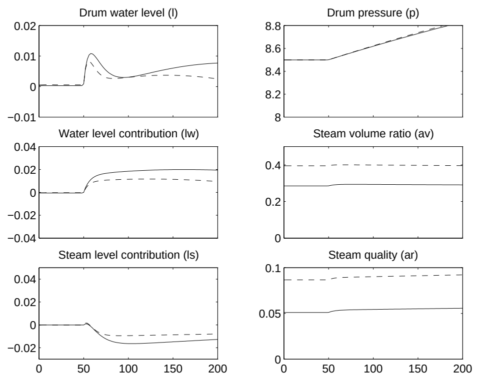
Step in Steam Flow Rate



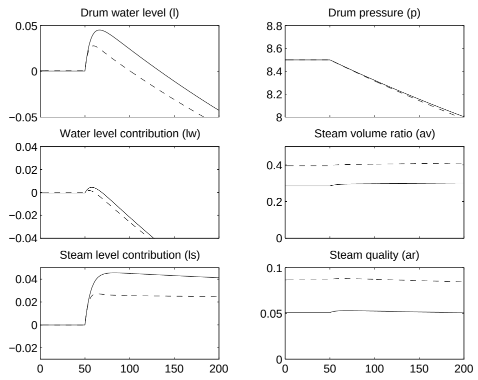
Step in Feedwater Flow Rate



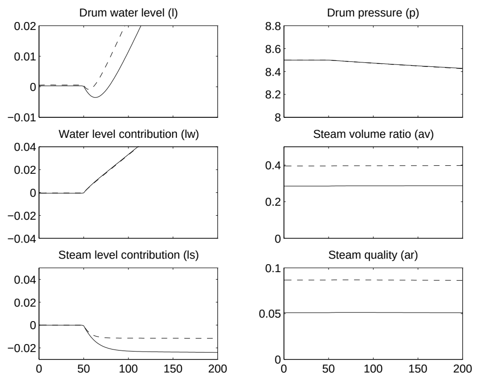
Step in Fuel Flow Rate Medium and High



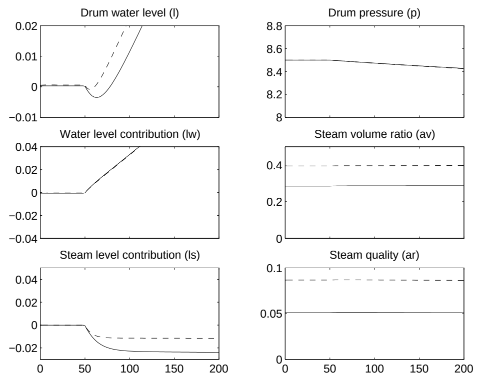
Step in Steam Flow Rate Medium and High



Step in Feedwater Flow Rate Medium and High



Step in Feedwater Temperature Medium and High

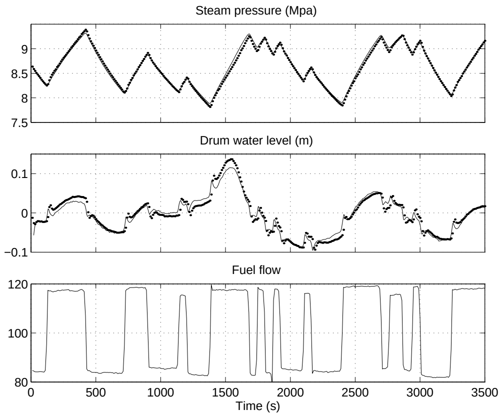


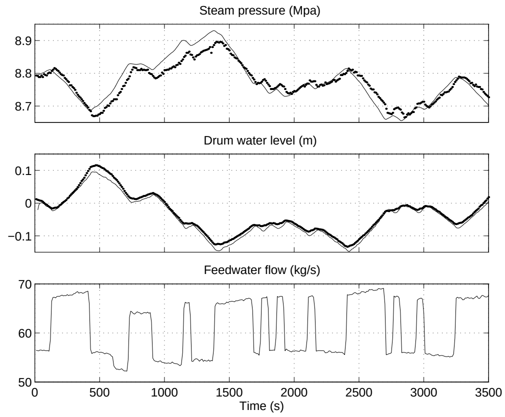
Lecture 9 - Boiler Modeling

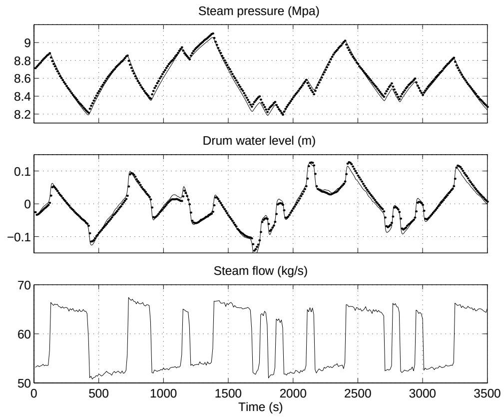
1. Introduction
2. Global Balance Equations
3. Steam Distribution
4. The Model
5. Simulation
6. Experiments
7. Conclusions

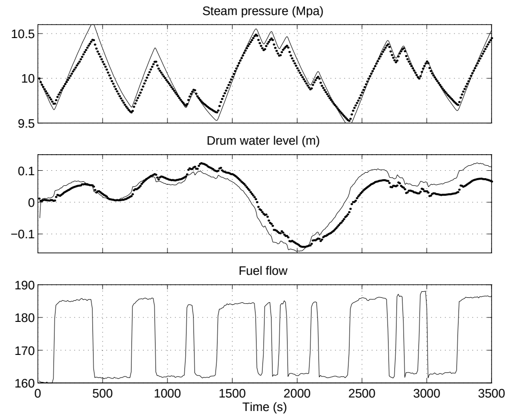
Experimental Validation

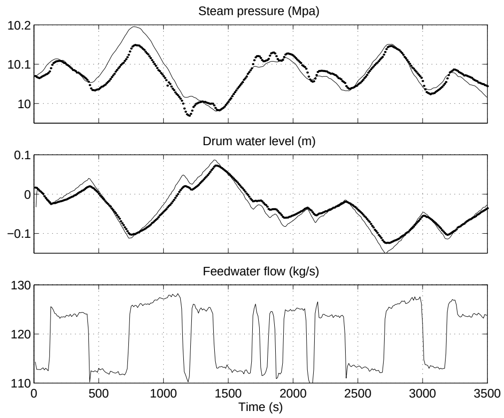
- Unique measurements Kalle Eklund (PhD number 1!)
- Controllers removed
- Very good excitations
- Many signals measured
- Why is this useful
- PRBS like perturbations
- Essentially the same input in all experiments
- Occasional manual interference
- Proper filtering of data
- Sampling rate 0.1 Hz

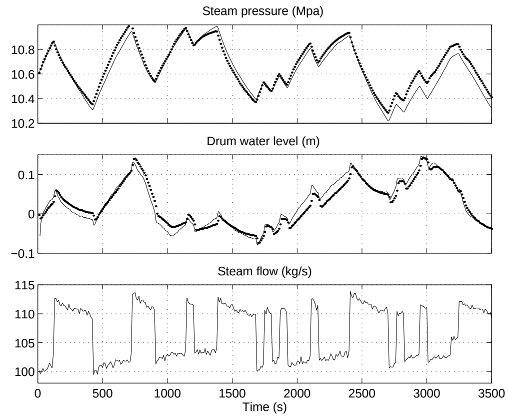


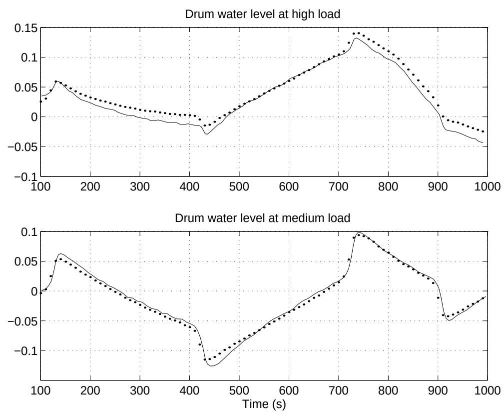


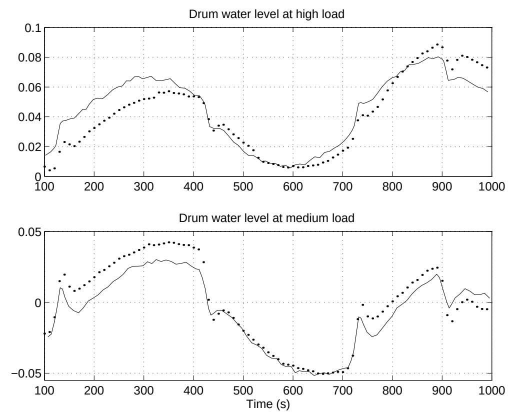


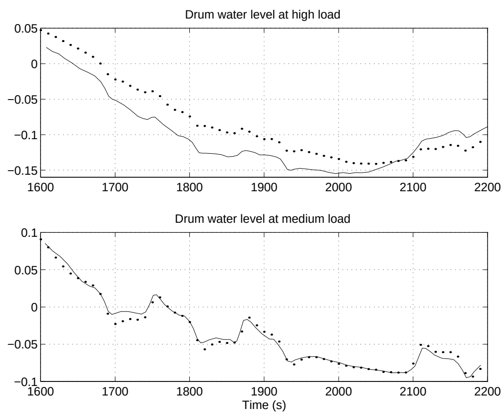












Lecture 9 - Boiler Modeling

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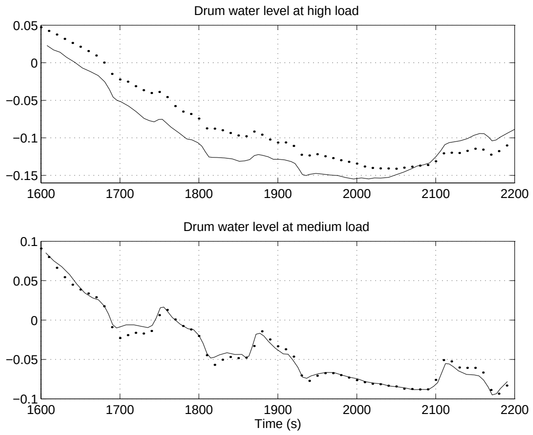
Conclusions

- Many different models have been explored
- Very good fit to data with 4th order model
- Rapid swell occurs
- Model simplification highly desired
  - Can we reduce the order?
  - What nonlinear terms can be neglected?
  - What parameters are important?
- New experiments
  - Higher sampling rates. We know what to look for
- Exploit power of Modelica
- Develop tools for model simplification

Introduction

- Sydkraft project
- Model based control
- Requirements
  - Capture essential dynamics
  - Reasonably simple
  - Parameters from construction data
- Properties of boilers
- A long story
- Kalle Eklund's thesis
- Rods visits
- First visit 1978
- IFAC Brussels 1988 - physics based
- IFAC Sydney 1993 - validation
- IFAC San Francisco 1996 - steam in drum
- Automatica Paper March 2000
- Where do we stand?
- New measurements
- Model simplification

Details of Data for Fuel Flow Changes



Conclusions

Where do we stand?

- Many ideas have been pursued
- Very good fit to data with 5th order model
- Rapid swell occurs
- Model simplification highly desired
  - Can we reduce the order?
  - What nonlinear terms can be neglected?
  - What parameters are important?
- Can we get simpler physical explanations
- New experiments
  - Sampling rate
  - We know what to look for
- Exploit power of Modelica
- Develop tools for model simplification