

## Compartment Models

K. J. Åström

1. Introduction
2. Compartment Models
3. Flow Systems
4. Measurement of Volumes and Flows
5. Summary
6. References

## Introduction

- ▶ Early work by Teorell and Widmark on propagation of alcohol in the body 1920
- ▶ Teorell coined the term compartment model around 1937
- ▶ Extensive application in pharmacokinetics Dost 1953
  - Models required for FDA approval of new drugs Sheppard and Householder 1953
- ▶ Pulse testing
  - Standard technique in ecological systems
  - Measurement of blood volume and blood flow in vessels
  - Extensive use in industry

## A Practical View

Doctors need simple models for the daily work. Key questions:

- ▶ How much drug should be administered?
- ▶ How should it be taken: inhalation, intravenously, intramuscularly, orally?
- ▶ How quickly will it act?
- ▶ How long will it act?

These questions all relate to the dynamics of propagation of drugs in the body.

## A Single Compartment Model

Consider a single compartment with volume  $V$  and flow  $q$ . Assume that the amount  $m$  is injected into the volume. The concentration is then given by

$$\frac{dc}{dt} = -\frac{q}{V}c$$

where the initial concentration is  $c(0) = m/V$ . Solving the differential equation we find that the concentration decays exponentially

$$c(t) = \frac{m}{V}e^{-qt/V}$$

The dynamics can be captured by two quantities

- ▶ Volume of distribution  $V [m^3]$
- ▶ Clearance  $q [m^3/s]$

The ratio  $q/V [s^{-1}]$  is called the elimination constant

## Volume of Distribution

Assume that there are two volumes  $V_1$  and  $V_2$  with concentrations  $c_1$  and  $c_2$  with strong interaction. Furthermore assume that the volume  $V_1$  is the one that is accessible to other compartments. If we represent the system with one compartment having the same mass, concentration  $c_1$  and volume  $V$  we find that

$$c_1V = c_1V_1 + c_2V_2$$

Hence

$$V = V_1 + \frac{c_2V_2}{c_1}$$

The volume  $V$  can be very large even if  $V_2$  is small if the ratio  $c_2/c_1$  is large

## Finding the Parameters

The concentration is given by

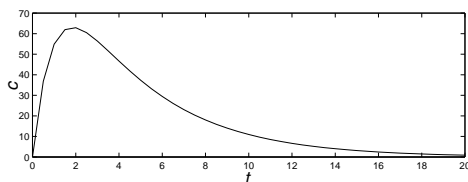
$$c(t) = \frac{m}{V}e^{-qt/V}$$

If the clearance and the volume of distribution are known it is easy to determine the concentration as a function of time  $t$  and dose  $m$ . The parameters  $q$  and  $V$  can be determined experimentally in the following way

- ▶ The volume of distribution can be determined from the dose  $m$  and the initial concentration  $c(0)$  using the formula  $V = m/c(0)$
- ▶ The elimination constant  $q/V$  can be determined by observing that  $\log c(t) = -qt/V$ . Plotting  $\log c(t)$  versus time gives a straight line with slope  $t$

## Back to Reality

Unfortunately the simple model does not agree with experiments. The concentration curve typically looks like this



which indicates that the first order model is too simplistic. In practice it is still used by neglecting the initial part of the curve and approximating the tail by a single exponential

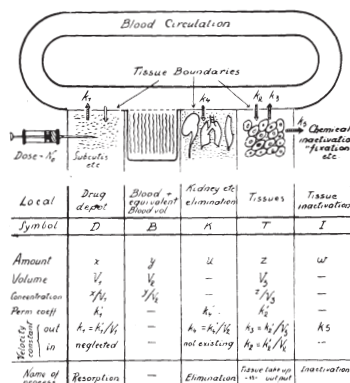
## Compartment Models

1. Introduction
2. Compartment Models
3. Flow Systems
4. Measurement of Volumes and Flows
5. Summary
6. References

## Compartment Models

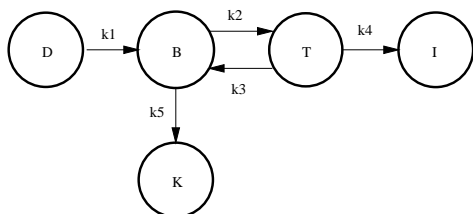
- ▶ Widmark 1920 Effect of alcohol on the human
- ▶ Teorell 1937 How drug injection propagates in the body
- ▶ Pharmacokinetics
- ▶ A special class of linear dynamical systems
- ▶ Nowadays required for FDA registration of medicines
- ▶ Advanced dosage (2 aspirin 3 times per day)
- ▶ Indirect measurements of volume and flow
- ▶ Use of radioactive tracers

## Teorell's Model



## Propagation of Drugs in the Body

T. Teorell (1937) Kinetics of Distribution of Substances Administered to the Body I and II. Arch. Int. de Pharm. et de Therapies 57(205-225), 57(226-240).



## A Mathematical Model

- ▶ Consider the body as a collection of compartments.
- ▶ Rate of transport between two compartments is proportional to the difference in concentration. For a system with two compartments we have

$$\frac{dx_1}{dt} = k(c_2 - c_1) = k\left(\frac{x_2}{V_2} - \frac{x_1}{V_1}\right) = -k_1x_1 + k_2x_2$$

$$\frac{dx_2}{dt} = k(c_1 - c_2) = k\left(\frac{x_1}{V_1} - \frac{x_2}{V_2}\right) = k_1x_1 - k_2x_2$$

where \$x\_1\$ is the number of molecules in compartment 1, and \$V\_1\$ its volume, and \$c\_1\$ the concentration in compartment 1, etc.

- ▶ Nonlinear transfer mechanism can also be used
- ▶ Notice that this particular model is closed!

## Two Ways to Write the Models

In terms of the extensive variable mass \$x\$ as

$$\frac{dx_1}{dt} = -\frac{k}{V_1}x_1 + \frac{k}{V_2}x_2$$

$$\frac{dx_2}{dt} = \frac{k}{V_1}x_1 - \frac{k}{V_2}x_2$$

or in terms of the intensive variable concentration \$c\$

$$\frac{dc_1}{dt} = -\frac{k}{V_1}c_1 + \frac{k}{V_1}c_2 = k_1(c_1 - c_2)$$

$$\frac{dc_2}{dt} = \frac{k}{V_2}c_1 - \frac{k}{V_2}c_2 = k_2(c_2 - c_1)$$

Notice structure of equations and coefficients

$$\frac{dx}{dt} = \begin{pmatrix} -k/V_1 & k/V_2 \\ k/V_1 & -k/V_2 \end{pmatrix} x, \quad \frac{dc}{dt} = \begin{pmatrix} -k_1/V_1 & k_1/V_1 \\ k_2/V_2 & -k_2/V_2 \end{pmatrix} c$$

## Left Eigenvalues and Invariants

$$\frac{dx}{dt} = Ax = \begin{pmatrix} -k/V_1 & k/V_2 \\ k/V_1 & -k/V_2 \end{pmatrix} x$$

We have

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} -k/V_1 & k/V_2 \\ k/V_1 & -k/V_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

A left eigenvector has zero eigenvalue. Physical interpretation

The audience is thinking!

## Conclusion

A left eigenvector \$v = (1 \ 1)\$ with eigenvalue \$\lambda = 0\$ to the matrix \$A\$ implies that

$$v \frac{dx}{dt} = \frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$$

Hence

$$x_1 + x_2 = \text{constant}$$

conservation of mass!

## Right Eigenvalues and Eigenvectors

$$\frac{dc}{dt} = Ac = \begin{pmatrix} -k/V_1 & k/V_1 \\ k/V_2 & -k/V_2 \end{pmatrix} c$$

We have

$$\begin{pmatrix} -k/V_1 & k/V_2 \\ k/V_2 & -k/V_2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

A right eigenvector has zero eigenvalue. Physical interpretation?

The audience is thinking!

## Conclusion

A right eigenvector  $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  with eigenvalue  $\lambda = 0$  to the matrix  $A$  implies that if the initial condition is

$$c(0) = v$$

it follows that

$$\frac{dc}{dt} = Ac$$

and we have  $Av = 0$  and the solution

$$x(t) = v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

is an equilibrium

## The General Case of the Mass Balance

The general case of the mass balance is

$$\frac{dx_i}{dt} = \sum_{j \neq i} k_{ij}x_j - \sum_{i \neq j} k_{ji}x_i + u_i = \sum_{j \neq i} k_{ij}x_j - k_{ij}x_i + u_i$$

where  $x_i$  is the mass in compartment  $i$ , and  $k_{ij}$  is the rate constant for transport from compartment  $j$  to compartment  $i$ .

Notice that  $k_{ij} \geq 0$  when  $i \neq j$ .

## An Alternative Representation

The equation can also be written as

$$\frac{dx}{dt} = Ax + Bu$$

with

$$a_{ij} \geq 0 \text{ for } i \neq j, \quad a_{ii} \leq 0, \quad |a_{ii}| \geq \sum_{j \neq i} a_{ij}, \quad b_i \geq 0$$

Notice that we can always obtain a system with

$$\sum a_{ij} = 0$$

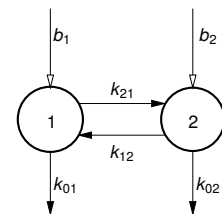
by including sources and sinks as extra compartments (a closed system)

## Graphical Representation

Mathematical model mass balance

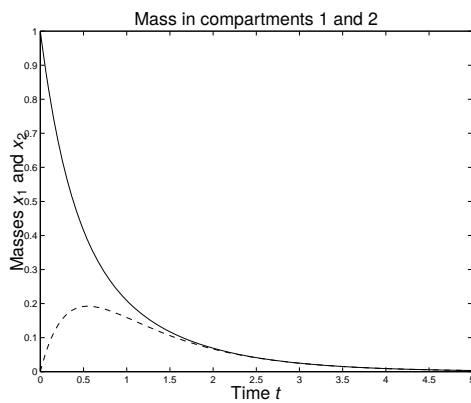
$$\frac{dx_1}{dt} = -(k_{01} + k_{21})x_1 + k_{12}x_2 + b_1u$$

$$\frac{dx_2}{dt} = k_{21}x_1 - (k_{02} + k_{12})x_2 + b_2u$$

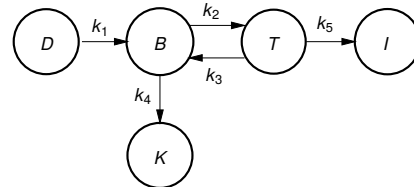


Assume the following numerical values  $k_{12} = k_{21} = k_{01} = k_{02} = 1$ . What happens if an impulse is injected to compartment 1.

## Results

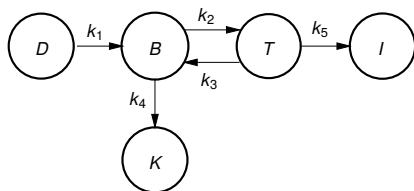


## Teorells Model



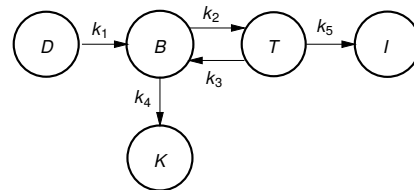
- D Tissue where drug is injected
- B Blood
- T Tissue where drug should work
- K Kidney
- I Drug inactivation

## Teorells Model



$$A = \begin{pmatrix} -k_1 & 0 & 0 & 0 & 0 \\ k_1 & -(k_2 + k_4) & k_3 & 0 & 0 \\ 0 & k_2 & -(k_3 + k_5) & 0 & 0 \\ 0 & k_4 & 0 & 0 & 0 \\ 0 & 0 & k_5 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

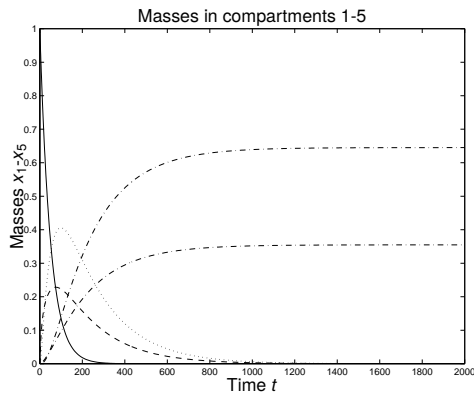
## A Simplified Model



$$A = \begin{pmatrix} -k_1 & 0 & 0 \\ k_1 & -(k_2 + k_4) & k_3 \\ 0 & k_2 & -(k_3 + k_5) \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1/V_b & 0 \\ 0 & 0 & 1/V_t \end{pmatrix}, \quad D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

## Results



## Some Interesting Problems

- ▶ Modeling: Determine number of compartments and transfer coefficients
- ▶ Effectiveness of the drug. How much comes to the right organ?
- ▶ Use of tracers
- ▶ Patient to patient variations
- ▶ Dosage: Assume that a certain concentration of drug is desired. How should it be administered.
- ▶ Discrete or continuous injection
- ▶ Tablets versus injections
- ▶ Nonlinear transfer mechanisms

## General Properties

- ▶ Compartment models are linear systems with special properties
- ▶ Related to positive matrices (Frobenius theorem)
  - All elements except the diagonal are positive
- ▶ Compartment systems are always stable (possibly an eigenvalue  $\lambda = 0$ )
- ▶ With some technical assumptions (irreducibility) it can be concluded that:
  - ▶ All eigenvalues of the system matrix  $A$  are in the left half plane or at the origin
  - ▶ For closed systems one eigenvalue is at the origin
  - ▶ For open systems all eigenvalues are in the left half plane

## An Example

Consider the simple cyclic system

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

The characteristic polynomial is

$$\det \lambda I - A = (\lambda + 1)^3 - 1 = \lambda(\lambda^2 + 3\lambda + 3)$$

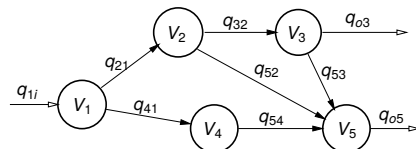
The eigenvalues are  $\lambda = 0$ , and  $\lambda = -3/2 \pm i\sqrt{3}/2$ . This system has complex eigenvalues and consequently oscillatory behavior.

## Compartment Models

1. Introduction
2. Compartment Models
3. Flow Systems
4. Measurement of Volumes and Flows
5. Summary
6. References

## Flow Systems

A collection of tanks with perfect mixing connected by pipes



Problems of measuring flows and volumes using tracers will now be investigated. Typical applications are

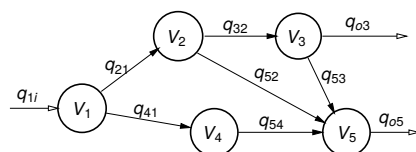
- ▶ Biomedical
- ▶ Industrial

## Classification of Flow Systems

- ▶ Open
- ▶ Closed
- ▶ Completely connected
- ▶ Catenary
- ▶ Circular
- ▶ Mammilar
- ▶ Symmetric

## Flow Systems

A collection of tanks with perfect mixing connected by pipes



$V_i$  Volume [ $m^3$ ] of  $i$ th tank

$q_{kl}$  Flow rate [ $kg/m^3$ ] from tank  $l$  to tank  $k$

$q_{ki}$  Inflow [ $kg/m^3$ ] to tank  $k$

$q_{oi}$  Outflow [ $kg/m^3$ ] from tank  $l$

## The Tracer Equations

Let  $c_k$  be the concentration of a tracer substance in tank  $V_k$ . Assume that the tracer moves in the same way as the fluid. Then

$$V_k \frac{dc_k}{dt} = \sum_{l=1, l \neq k}^n q_{kl} c_k - \sum_{l=0, l \neq k}^n q_{lk} c_l + q_{kl} c_l$$

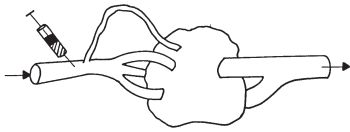
where

$$\sum_{l=1, l \neq k}^n q_{kl} + q_{kl} = 0$$

## Compartment Models

1. Introduction
2. Compartment Models
3. Flow Systems
4. Measurement of Volumes and Flows
5. Summary
6. References

## Tracer Experiments



Idea: Inject a tracer in the system. Measure tracer concentration at some point in the system. Determine volumes and flows. Compare with counting of animals in the wood!

## Tracer Propagation - Example 1

Consider a tank with volume  $V$ . Assume that there is a flow with rate  $q$  through the tank. Inject a tracer with concentration  $c_{in}$  at the inlet. Assume that the tracer mixes perfectly with the fluid. A mass balance gives

$$V \frac{dc}{dt} = q(c_{in} - c)$$

where  $c$  is the concentration in the tank and in the outlet. The relation between  $c$  and  $c_{in}$  is a dynamical system with impulse response

$$g(t) = e^{-\frac{qt}{V}}$$

and transfer function

$$G(s) = \frac{1}{1 + sV/q}$$

## Tracer Propagation - Example 2

Consider a pipe with volume  $V$ . Assume that there is a flow with rate  $q$  through the pipe. Inject a tracer with concentration  $c_{in}$  at the inlet. Assume that the tracer mixes perfectly with the fluid. The concentration  $c_{out}$  at the outlet is then given by

$$c_{out}(t) = c_{in}\left(t - \frac{V}{q}\right)$$

The relation between  $c_{out}$  and  $c_{in}$  is a dynamical system with the impulse response

$$g(t) = \delta\left(t - \frac{V}{q}\right)$$

and the transfer function

$$G(s) = e^{-sq/V}$$

## Tracer Propagation - General Properties

Consider a flow system with one inlet and one outlet. The propagation of a tracer through the system is a linear dynamical system with one input and one output. Let  $g$  be the impulse response of the system. The impulse response has the properties

$$g(t) \geq 0$$

$$\int_0^{\infty} g(t) dt = 1$$

The first equation says that the outlet concentration is never negative. The second equation says that all injected tracer must finally come out.

## The Stewart Hamilton Equation

Consider a flow system with one inlet and one outlet. Let the inlet and outlet flows be  $q$  and let the volume of the system be  $V$ . Furthermore let  $g$  be the impulse response of the system. Then

$$\frac{V}{q} = \int_0^{\infty} tg(t) dt = T_{ar}$$

This formula has been used extensively in biology, ecology and engineering to determine volumes and flows. The quantity  $T_{ar}$  is called the average residence time.

## Proof

The impulse response  $g(t)$  can be interpreted as a probability density function for the time it takes a particle to pass through the system. Consider a collection of particles that enter the system at time  $t$ . The quantity  $g(t)dt$  is the fraction of particles that enter the system in the time interval  $(t, t + dt)$ . These particles have traversed the volume  $qt$ . The total volume visited by all particles is thus

$$V = \int_0^{\infty} qtg(t) dt = q \int_0^{\infty} tg(t) dt$$

Dividing by  $q$  gives the Stewart-Hamilton equation.

### Examples

Consider a tank having volume  $V$  [ $m^3$ ] and flow  $q$  [ $m^3/s$ ] with perfect mixing. The impulse response is

$$g(t) = \frac{q}{V} e^{-qt/V}$$

Integration gives

$$\int_0^{\infty} g(t) dt = \int_0^{\infty} \frac{q}{V} e^{-qt/V} dt = 1$$

Furthermore

$$\int_0^{\infty} tg(t) dt = \int_0^{\infty} t \frac{q}{V} e^{-qt/V} dt = \frac{V}{q}$$

Hence if the impulse response is measured and we know the flow the volume can be computed.

### Computing Average Residence Times

Transfer functions are very convenient to use when computing average residence times. We have

$$G(s) = \int_0^t e^{-st} g(t) dt$$

Differentiating this with respect to  $s$  we get

$$G'(s) = - \int_0^t te^{-st} g(t) dt$$

Hence

$$T_{ar} = \int_0^t te^{-st} g(t) dt = -G'(0)$$

The average residence time  $T_{ar}$  is a useful characteristic.

### Examples

Tank with perfect mixing

$$G(s) = \frac{1}{1 + sV/q}$$

$$G'(s) = - \frac{1}{(1 + sV/q)^2} \frac{V}{q}$$

$$T_{ar} = -G'(0) = \frac{V}{q}$$

Pipe with perfect mixing

$$G(s) = e^{-sV/q}$$

$$G'(s) = - \frac{V}{q} e^{-sV/q}$$

$$T_{ar} = -G'(0) = \frac{V}{q}$$

### Series Connection of Tanks

The transfer function for a series connection is

$$G(s) = G_1(s)G_2(s)$$

Hence

$$G'(s) = G'_1(s)G_2(s) + G_1(s)G'_2(s)$$

$$T_{ar} = G'(0) = T_{ar1} + T_{ar2}$$

Since  $G_i(0) = 1$

### Measurement of Volume and Flow

Consider a flow system with volume  $V$  [ $m^3$ ] and through flow  $q$  [ $m^3/s$ ]. Inject a given amount  $m$  of a tracer rapidly. If the injection time is so short that the injection can be regarded as an impulse. Let  $c(t)$  [ $kg/s$ ] be the concentration of the substance at the outlet. We have

$$q \int_0^{\infty} c(t) dt = M$$

The flow can thus be determined by

$$q = \frac{M}{\int_0^{\infty} c(t) dt}$$

The quantity  $q$  which has dimension [ $m^3/s$ ] is called clearance

### Measurement of Volume and Flow

We have

$$q = \frac{M}{\int_0^{\infty} c(t) dt}$$

It follows from the Stewart-Hamilton equation that

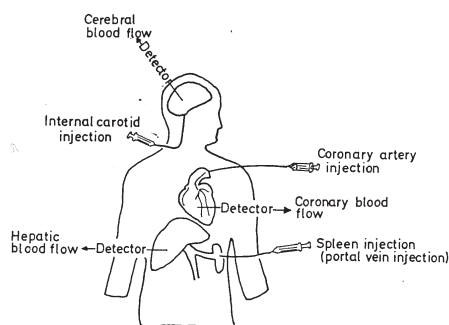
$$\frac{V}{q} = \frac{\int_0^{\infty} tc(t) dt}{\int_0^{\infty} c(t) dt}$$

Hence

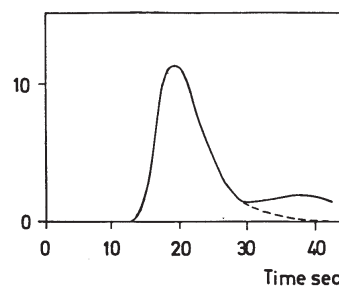
$$V = \frac{M \int_0^{\infty} tc(t) dt}{\left(\int_0^{\infty} c(t) dt\right)^2}$$

Practical problems in computation of the infinite integrals

### Many Examples

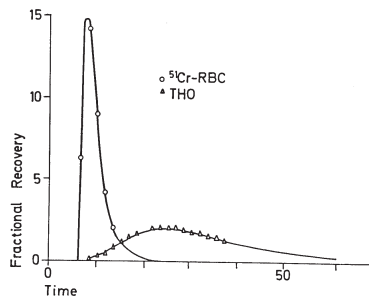


### Impulse Response of the Heart



Notice the effect of recirculation.

### Choice of Tracer is Important



$Cr^{51}$  stays vascular, but THO penetrates extracellular

### Volume of Distribution and Virtual Flow

In biological systems it is desirable to have a concept of volume for systems with very complicated flow patterns. The volume of distribution is such a measure. Assuming that the total amount of substance is  $m$  [kg] and that the system is in equilibrium. The concentration is then the same in all compartments and the volume of distribution is defined as

$$V_{dist} = \frac{m}{c}$$

This formula is often used in medicine to measure volumes. To apply it it is essential that the system is in equilibrium.

Since the concentration not necessarily is constant the volume is not necessarily equal to the true physical volume it is called *volume of distribution or apparent volume*.

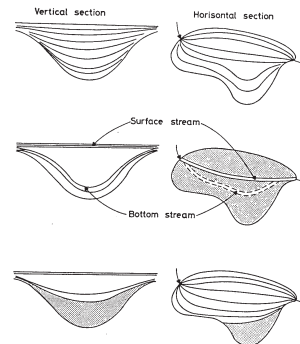
### The Concept of Clearance

There are many different mechanisms that govern transport from one compartment to another. It is practical to have a simple way to describe removal of a substance from a compartment. Clearance or volumetric flow is such a measure. For a volume  $V$  with through-flow  $q$  [ $m^3/s$ ] the clearance is simply  $q$ . Clearance can also be defined as

$$q = \frac{w}{c}$$

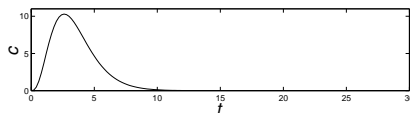
where  $w$  [kg/s] is the mass flow rate and  $c$  [kg/ $m^3$ ] the concentration.

### Applications in Ecology

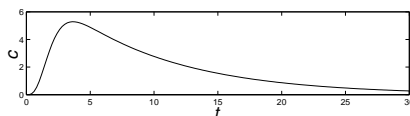


### Applications in Industry

Tank volume  $4000 m^3$ , flow  $350 m^3/h$ , average residence time  $T_{ar} = 11.4$ .  
 Tank with 2 stirrers:  $T_{ar} = 3.5 h$ ,  $V_e = 3.5 \times 350 = 1225$



Tank with 4 stirrers:  $T_{ar} = 10.6 h$ ,  $V_e = 10.6 \times 350 = 3710$



### Finding the Parameters of the System

We will first determine the input output relation between the injection and the concentration. The system is characterized by the matrices

$$A = \begin{pmatrix} -k_0 - k_1 & k_1 \\ k_2 & -k_2 \end{pmatrix}, \quad B = \begin{pmatrix} k_0 \\ 0 \end{pmatrix}, \quad C = (1 \ 0)$$

The transfer function for the system is

$$G(s) = C(sI - A)^{-1}B = (1 \ 0) \begin{pmatrix} s + k_0 + k_1 & -k_1 \\ -k_2 & s + k_2 \end{pmatrix}^{-1} \begin{pmatrix} k_0 \\ 0 \end{pmatrix}$$

$$= \frac{k_0(s + k_2)}{s^2 + (k_0 + k_1 + k_2)s + k_0k_2}$$

This calculation shows that the parameter combinations  $k_2$ ,  $k_0 + k_1 + k_2$  and  $k_0k_2$  can be determined which implies that all rate constants can be computed from the experiment

### Finding the Parameters of the System

Consider an experiment where 10 g has been injected in compartment 1 and the following concentration has been measured in compartment 1.

$$c(t) = A_1 e^{-\lambda_1 t} + A_2 e^{-\lambda_2 t} = 0.38e^{-1.65t} + 0.18e^{-0.182t}$$

Taking Laplace transforms we find

$$C(s) = \frac{A_1}{s + \lambda_1} + \frac{A_2}{s + \lambda_2} = \frac{s(A_1 + A_2) + A_1\lambda_2 + A_2\lambda_1}{s^2 + (\lambda_1 + \lambda_2)s + \lambda_1\lambda_2}$$

Identifying this with the expression for  $G(s)$  we find

$$k_2 = \frac{A_1\lambda_2 + A_2\lambda_1}{A_1 + A_2}$$

$$k_0 + k_1 + k_2 = \lambda_1 + \lambda_2$$

$$k_0k_2 = \lambda_1\lambda_2$$

### Finding the Parameters of the System

$$k_2 = \frac{A_1\lambda_2 + A_2\lambda_1}{A_1 + A_2}$$

$$k_0 + k_1 + k_2 = \lambda_1 + \lambda_2$$

$$k_0k_2 = \lambda_1\lambda_2$$

Solution

$$k_0 = \frac{\lambda_1\lambda_2(A_1 + A_2)}{A_1\lambda_2 + A_2\lambda_1}$$

$$k_1 = \frac{A_1A_2(\lambda_1 - \lambda_2)^2}{(A_1 + A_2)(A_1\lambda_2 + A_2\lambda_1)}$$

$$k_2 = \frac{A_1\lambda_2 + A_2\lambda_1}{A_1 + A_2}$$

Inserting the numerical values  $A_1 = 0.38$ ,  $A_2 = 0.18$ ,  $\lambda_1 = 1.65$  and  $\lambda_2 = 0.182$  give  $k_0 = 0.459$ ,  $k_1 = 0.719$  and  $k_2 = 0.654$ .

## Determining Volumes and Flows

Initial concentration  $c_1(0) = 0.38 + 0.18 = 0.56 \text{ [kg/m}^3\text{]}$ , injected amount 0.010 kg. Assuming perfect mixing

$$V_1 = \frac{0.010}{0.56} = 0.0179 \text{ [m}^3\text{]}$$

The outflow from volume  $V_1$  is  $q_0 = V_1 k_0 = 0.008201$ , furthermore we have

$$V_2 = \frac{k_1}{k_2} V_1 = 0.0196$$

The total volume is thus  $V_1 + V_2 = 0.0375$ . It can also be computed using the Stewart-Hamilton equation. This gives

$$V_t = q_0 T_{ar} = q_0 \frac{A_1 \lambda_1^{-2} + A_2 \lambda_2^{-2}}{A_1 \lambda_1^{-1} + A_2 \lambda_2^{-1}} = 0.0375$$

## Compartment Models

1. Introduction
2. Compartment Models
3. Flow Systems
4. Measurement of Volumes and Flows
5. Summary
6. References

## Summary

- ▶ Compartment models
- ▶ An interesting special case of linear systems
- ▶ Many interesting properties
- ▶ Many applications industry, biology, medicine
- ▶ Measurement of "volumes" and "flows"
- ▶ The Stewart Hamilton Equation

## References

- Widmark, E. P. M and J. Tandberg (1925)  
Teorell, T (1937) Kinetics of distribution of substances administered to the body I and II *Archiv International Pharmacodynamie Therapie* 57, 205–225 and 57, 226-240.  
Sheppard, C. W. (1948) The theory of transfers within a multi-compartment system using isotopic tracers. *J. Appl. Phys.* 19, 70-76.  
Riggs, D. S. (1963) *The Mathematical Approach to Physiological Problems*. MIT Press, Boston, MA.  
Dost, F. H. (1968) *Grundlagen der Pharmakokinetik*. T. Thieme. Stuttgart.

## References

- Atkins G. L. (1969) *Multicompartment Models for Biological Systems*, Methuen, London.  
Bellman, R. and K. J. Åström (1970) On structural identifiability. *Math. Biosciences.* 7, 329-339.  
Jacquez, J. A. (1972) *Compartmental Analysis in Biology and Medicine*. Elsevier, Amsterdam.  
Matis, J. H. B. C. Patten and G. C. White (eds) (1979) *Compartmental Analysis in Ecosystem Models*. Int. Co-operative Publishing House. Fairland, MD.  
Godfrey, G. (1982) *Compartment Models and their Application*. Academic Press New York.

## References

- Shargel, L. Andrew B. C. Yu (1999) *Applied Biopharmaceutics and Pharmacokinetics*.  
Rowland, M. Thomas N. Tozer (1995) *Clinical Pharmacokinetics : Concepts and Applications*  
Cobelli, C. D. Foster and G. Toffolo (2001) *Tracer Kinetics in Biomedical Research: From Data to Model*. Plenum Pub Corp; ISBN: 0306464276  
Carson, and C. Cobelli. (2001) *Introduction to Modeling in Physiology and Medicine* Academic Press