Fluid Dynamics Modeling

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1. Introduction
2. Review of Fluid Dynamics
3. Simple Water Tank
4. Simple Gas Tank
5. Tanks, Pipes and Turbines
6. Summary

Historical Remarks

- Hydroelectric power
- Control of dams and turbines
- Founded in civil engineering
  - A not so well recognized base of automatic control
  - Evangelisti (an IFAC founder) in Italy
  - Many others in civil engineering
  - Vattenfalls Älvkarleby Laboratory
- Interesting examples

A Modeling Methodology

- Cut a system into subsystems
- Write mass, momentum and energy balances for each subsystem
- State variables describe storage
- How accurate do we need to describe storage?
- The model format is differential algebraic equations
- Use object orientation to structure the system
- Let software (Modelica) handle bookkeeping and transformations
- Build component libraries

Modeling Check List

- Understand the process
- Representations
- Mathematical models
- Steady state properties
- Nonlinear dynamical models
- Linearization
- Approximation simplification
- Validation
- Librarization

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Review of Fluid Dynamics

Fluid dynamics is much more complicated than circuit theory. A prototype for physical modeling.

- Learn the basics
- A large complex field
- Consult the specialists
- Computational Fluid Dynamics (CFD)
- Culture clashes

Related fields

- Continuum Mechanics
- Fluid Mechanics
- Gas Dynamics
- Hydrology
- Rheology
- Field theory

Different Points of View

Equations are obtained by

- Lagrange: Follow a "fluid particle"
- Euler: Analyze what happens at a fixed point

Equations can be written in

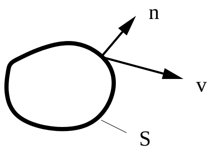
- Integral form
- Differential form

The Theoretical Body

- Long winded calculations (Navier, Stokes and Lamb)
- Vector analysis
  - div, grad, rot, ∇
  - rot works only in R^3
- Tensor calculus
  - Covariant and contravariant summation convention $a^{ij}b_j$
- Differential geometry
  - Nice and clean
  - Should be part of our basic education

Balance Equations

Mass balance (Continuity Equation)



$$\frac{\partial}{\partial t}\int_V \varrho dV + \int_S (\hat{n}\varrho v_{rel}) dS = 0$$

Momentum Balance

$$\frac{\partial}{\partial t}\int_V \varrho v dV + \int_S v_{abs}(\hat{n}\varrho v_{rel}) dS = \int_V F dV - \int_S p\hat{n} dS$$

Energy Balance Bernoullis Equation

$$\int_A^B \frac{\partial v}{\partial t} dr + \frac{1}{2}(v_B^2 - v_A^2) + \Omega_B - \Omega_A + \int_A^B \frac{dp}{\varrho} = 0$$

Mass Balance - The Continuity Equation

Integral form

$$\frac{\partial}{\partial t}\int_V \varrho dV + \int_S (\hat{n}\varrho v_{rel}) dS = 0$$

Gauss theorem fix control surface

$$\int_V \Big(\frac{\partial \rho}{\partial t} + \mathrm{div}(\varrho v)\Big) dV$$

Differential form

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \mathrm{div}(\varrho v) = 0$$
$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$
$$\nabla^T = \Big(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\Big)$$

Euler's Equations of Motion

Assume frictionless fluid with constant density
Integral form of momentum balance

$$\frac{\partial}{\partial t}\int_V \varrho v dV + \int_S v_{abs}(\hat{n}\varrho v_{rel}) dS = \int_V \varrho F dV - \int_S p\hat{n} ds$$

Differential form

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + (v \,\mathrm{grad}\,) v = F - \frac{1}{\varrho}\mathrm{grad}\, p$$
$$\frac{dv}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = F - \frac{1}{\rho}\nabla p$$

where

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

Navier Stokes Equation

Now consider effects of viscosity Navier (1827) and Stokes (1845)

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + (v \,\mathrm{grad}\,) v = F - \frac{1}{\varrho}\mathrm{grad}\, p + \frac{\lambda + \mu}{\varrho}\mathrm{grad}\,\mathrm{div}\, v + \frac{\mu}{\varrho}\Delta v$$
$$\frac{dv}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = F - \frac{1}{\rho}\nabla p + \frac{\lambda + \mu}{\rho}\nabla(\nabla \cdot \mathbf{v}) + \frac{\mu}{\rho}\Delta \mathbf{v}$$

where μ is the viscosity and λ the volume compression factor

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

Constitutive Equations

Compressible fluid

$$\frac{d\varrho}{\varrho} = -\kappa p(\rho)$$

where κ is the bulk compressibility.
For gases

$$p = p_0 \frac{\varrho}{\varrho_0}, \qquad \text{Isothermic}$$
$$p = p_0 \Big(\frac{\varrho}{\varrho_0}\Big)^{\gamma}, \qquad \text{Adiabatic}$$

where $\gamma = C_p/C_v$

Dimension Free Parameters

Extensively used in modeling

- Find suitable variables to express physical relations
- Presentation of empirical data
- Designing scale experiments

  Ship resistance the Froude's number (1970)

  $$Fr = \frac{v^2}{lg}$$

  gives the ration of inertial forces to gravity
- Preliminary (crude) model validation
- Judge what effects are important

Reynolds Number

Navier-Stokes equation

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + (v \,\mathrm{grad}\,) v = F - \frac{1}{\varrho}\mathrm{grad}\, p + \frac{\lambda + \mu}{\varrho}\mathrm{grad}\,\mathrm{div}\, v + \frac{\mu}{\varrho}\Delta v$$

Introduce $\bar{v} = v/v_0, \bar{x} = x/x_0, \bar{t} = v_0 t/x_0, \bar{F} = x_0 F/v_0^2, \bar{p} = p/(v_0 x_0)^2$.
The equation then becomes

$$\frac{\partial \bar{v}}{\partial \bar{t}} + (\bar{v} \,\mathrm{grad}\,)\bar{v} = \bar{F} - \frac{1}{\varrho}\mathrm{grad}\,\bar{p} + \frac{\mu}{v_0 x_0 \varrho}\Delta \bar{v}$$

The Reynolds number (ratio of inertial and friction forces)

$$Re = \frac{\varrho v d}{\eta}$$

tells when viscosity is important

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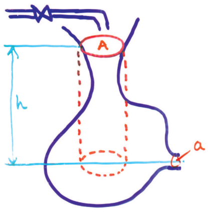
A Simple Water Tank

How do level h and outflow q_{out} depend on the inflow q_{in}?
Assume: Constant density

$$\frac{dV}{dt} = q_{in} - q_{out} \quad \text{Massbalance}$$
$$V = \int_0^h A(h)dh \quad \text{Geometry}$$
$$q_{out} = a\sqrt{2gh} \quad \text{Energybalance}$$



What parameters are relevant?
How do we determine them?

Analysis and Simplification

Choosing h as a state variable we find

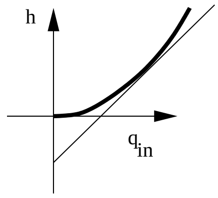
$$\frac{dh}{dt} = \frac{1}{A(h)}(q_{in} - a\sqrt{2gh})$$
$$q_{out} = a\sqrt{2gh}$$

One function $A(h)$ and one parameter a.

Steady state relation

$$q_{out} = q_{in}$$
$$h = \frac{q_{in}^2}{2ga^2}$$

Not influenced by A!



A Difficulty

The equation

$$\frac{dh}{dt} = \frac{1}{A(h)}(q_{in} - a\sqrt{2gh})$$

Equation not Lipschitz for $h = 0$! Trouble!
When water level is below the upper part of the hole we get

$$q_{out} = \int_0^h \sqrt{2gx}\, dA(x)$$

Rectangular cross section of width b give $dA(x) = bdx$, hence

$$q_{out} = \int_0^h \sqrt{2gx} bdx = \frac{2}{3} bh\sqrt{2gh}$$

Linearization

Equations

$$\frac{dh}{dt} = \frac{1}{A(h)}(q_{in} - a\sqrt{2gh})$$
$$q_{out} = a\sqrt{2gh}$$

Transfer functions

$$\frac{H}{Q_in} = \frac{2h_0}{q_0}\frac{1}{1+sT}$$
$$\frac{Q_{out}}{Q_in} = \frac{1}{1+sT}$$

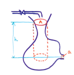
Linearized equation

$$\frac{d\delta h}{dt} = -\frac{q_0}{2A(h_0)h_0}\delta h + \frac{1}{A(h_0)}\delta q_{in}$$
$$\delta q_{out} = \frac{q_0}{2h_0}\delta q_{in}$$

Static gain $\frac{2h_0}{q_0}$.

Time constant $T = \frac{2Ah_0}{q_0}$

- Physical interpretation



Solving the Equation

For zero inflow we have

$$\frac{dh}{dt} = -\frac{a}{A}\sqrt{2gh}$$

The solution is

$$h = \left(\sqrt{h_0} - \frac{a}{A}\sqrt{g/2t}\right)^2 = h_0\left(1 - \frac{t}{T}\right)^2$$
$$q_{out} = a\sqrt{2gh_0}\left(1 - \frac{t}{T}\right) = q_0\left(1 - \frac{t}{T}\right)$$

Summary of Simple Water Tank

- Simple prototype modeling
- Equations difficult to solve analytically even in this simple case (cf Vannevar Bush)
- Mathematical conditions (Lipschitz good indicators)
- Linearization and steady state solutions helpful for insight
- Special cases useful to validate results
- Physical interpretation of parameters useful
- Think about good transformations

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The Simple Gas Tank

We will now look at the same problem as the simple tank but we will change from an incompressible to a compressible fluid. The only thing that changes is thus the constitutive equations. The behavior will however be very different.

Constitutive Equations

Isothermic changes, the Ideal Gas Law

$$pV = nRT$$

$$p = p_0 \frac{\rho}{\rho_0}$$

Adiabatic state changes

$$p = p_0 \left(\frac{\rho}{\rho_0}\right)^{\kappa}$$

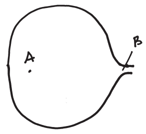
where $\kappa = c_p/c_v$

Fluid	Air	He	CO_2
κ	1.4	1.66	1.30

Flow Through a Nozzle

Energy balance

$$\frac{v^2}{2} - \frac{v_A^2}{2} + \int_A^B \frac{dp}{\rho} = 0$$



Constitutive equation

$$p = p_A \left(\frac{\rho}{\rho_0}\right)^{\kappa}$$

$$dp = \kappa p_A \left(\frac{\rho}{\rho_0}\right)^{\kappa-1} \frac{d\rho}{\rho}$$

Hence

$$v_B^2 = \frac{2\kappa}{\kappa-1}\frac{p_A}{p_B}\left(1 - \left(\frac{\rho_B}{\rho_A}\right)^{\frac{\kappa-1}{\kappa}}\right)$$

Velocity

$$v_B = \sqrt{\frac{2\kappa}{\kappa-1}\frac{p_A}{p_B}\left(1 - \left(\frac{\rho_B}{\rho_A}\right)^{\kappa-1}\right)}$$

$$v_B = \sqrt{\frac{2\kappa}{\kappa-1}\frac{p_A}{p_B}\left(1 - \left(\frac{p_B}{p_A}\right)^{\frac{\kappa-1}{\kappa}}\right)}$$

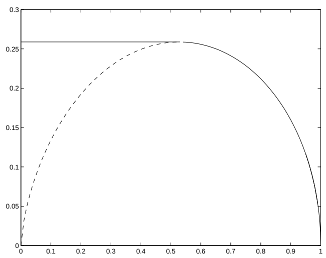
Mass Flow Rate

$$W_B = a\sqrt{\frac{2\kappa p_A \rho_A}{\kappa-1}\left(\left(\frac{p_B}{p_A}\right)^{\frac{2}{\kappa}} - \left(\frac{p_B}{p_A}\right)^{\frac{\kappa+1}{\kappa}}\right)}$$

Critical pressure

$$\frac{p_c}{p_A} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$$

For air $p_c/p_A = 0.53$.
Speed of sound at critical pressure



p_B/p_A

Comment

When the flow rate equals the speed of sound the communication from down stream to upstream is broken. For air this happens approximately when $p_A > p_B$.
Blandaren: Om man springer fortare an ljuset svartnar det för ögonen!

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Pressure Drop in Pipes

Laminar Flow Hagen Poiseuille 1840

$$\Delta p = 8\eta\frac{\ell v}{r^2} + 2\frac{\varrho v^2}{2}$$

General expression

$$\Delta p = f\frac{\ell}{d}\frac{1}{2}\varrho v^2$$

Effects on inlet shape on pressure drop

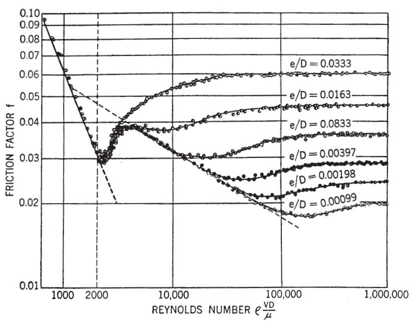


Fig. 8-5. Plot of Nikuradse's data on flow in artificially roughened pipes.

Tank with Outlet Pipe

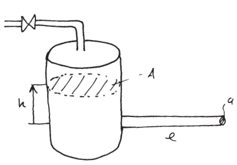
Mass storage in tank and momentum storage in pipe
Energy balance:

$$\int_A^B \frac{\partial v}{\partial t} dr + \frac{1}{2}(v_B^2 - v_A^2) + \Omega_B - \Omega_A + \int_A^B \frac{dp}{\varrho} = 0$$

Constant velocity v along the pipe turbulent flow

$$\ell\frac{dv}{dt} + \frac{1}{2}(v^2 - 0) - gh + f\frac{l}{d}\frac{v^2}{2} = 0$$

$$\ell\frac{dv}{dt} + \frac{1+c}{2}v^2 - gh = 0$$



where $c = f\frac{\ell}{d}$

Tank with Pipe and Turbulent Flow

Summary of equations

$$\frac{dh}{dt} = -\frac{a}{A}v + \frac{1}{A}q_{in}$$
$$\frac{dv}{dt} = \frac{g}{\ell}h - \frac{1+c}{2\ell}v^2$$
$$q_{out} = av$$

Linearize

$$\frac{d\delta h}{dt} = -\frac{a}{A}\delta v + \frac{1}{A}\delta q_{in}$$
$$\frac{d\delta v}{dt} = \frac{g}{\ell}\left(\delta h - \frac{2h}{v}\delta v\right)$$
$$\delta q_{out} = a\delta v$$

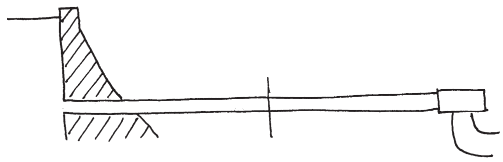
Stationary solutions

$$h = \frac{1+c}{2a^2 g}q_{in}^2$$
$$v = \frac{1}{a}q_{in}$$
$$q_{out} = q_{in}$$

Frequency and damping

$$\omega = \sqrt{\frac{ag}{A\ell}}$$
$$\zeta = \frac{1+c}{2a}\sqrt{\frac{A}{ag\ell}}q_{in} = \frac{h}{v}\sqrt{\frac{Ag}{a\ell}}$$

Water Turbine



How does turbine power depend on valve opening?
Water hammer is a phenomenon that can occur in any piping system where valves are used to control the flow of liquids or steam. Water hammer is the result of a pressure surge, or high-pressure shockwave that propagates through a piping system when a fluid in motion is forced to change direction or stop abruptly.

Equations

Bernoulli

$$l\frac{dv}{dt} + \frac{1}{2}\left(v_u^2 - 0\right) - gh + \frac{c}{2}v^2 = 0$$
$$a_u v_u = av$$
$$P = \frac{1}{2}\varrho a_u v_u^3 = \frac{1}{2}\varrho\frac{a^3}{a_u^2}v^3$$

Add mass balance:

$$\frac{dh}{dt} = -\frac{a}{A}v + \frac{1}{A}q_{in}$$
$$\frac{dv}{dt} = \frac{g}{\ell}h - \frac{a^2 + ca_u^2}{2\ell a_u^2}v^2$$
$$P = \frac{1}{2}\varrho\frac{a^3}{a_u^2}v^3$$

Stationary solutions

$$h = \frac{a^2 + ca_u^2}{2ga^2a_u^2}q_{in}^2$$

Assume Perfect Level Control

Assume that the inflow is used to keep the level constant and neglect friction. The model then becomes.

$$\frac{d\delta v}{dt} = -\frac{2gh}{\ell}\left(\frac{1}{v}\delta v - \frac{1}{a_u}\delta a_u\right)$$
$$\delta P = \frac{3P}{v}\delta v - \frac{2P}{a_u}\delta a_u$$

Transfer function

$$G_{\delta P\delta a_u} = \frac{P}{a_u}\frac{1-2sT}{1+sT}$$

where

$$T = \frac{\ell v}{2gh} = \frac{\ell v}{v_u^2} = \frac{\ell}{v_u}\frac{a_u}{a} == \frac{\ell}{v}\left(\frac{a_u}{a}\right)^2$$

Compressible Fluids Water Hammer

Constitutive equation: $\frac{d\varrho}{\varrho} = -\kappa p$

Scaled inertia term

$$v\,\mathrm{gr\bar{a}d}\,\mathrm{v} = \frac{v_0^2}{x_0}\frac{\partial\bar{v}}{\partial\bar{x}}$$

Scaled elastic forces

$$\frac{1}{\varrho}\mathrm{grad}\,p = \frac{1}{\kappa\varrho_0 x_0}\frac{\partial\bar{\varrho}}{\partial\bar{x}}$$

Ratio of inertia and elastic forces

$$M^2 = \frac{v_0^2 x_0\kappa\varrho_0}{x_0} = \kappa\varrho_0 v_0^2 = \left(\frac{v_0}{c}\right)^2$$

Compressible Fluids Water Hammer

Mass balance (Continuity Equation)

$$\frac{\partial v}{\partial t} + \mathrm{div}(\varrho v) = 0$$

Navier Stokes

$$\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = F - \frac{1}{\rho}\nabla p + \frac{\lambda+\mu}{\rho}\nabla(\nabla\cdot\mathbf{v}) + \frac{\mu}{\rho}\Delta\mathbf{v}$$

Constitutive equation

$$\frac{\partial p}{\partial x} = \frac{1}{\kappa\varrho_0}\frac{\partial\varrho}{\partial x}$$

Linearization

Neglect inertia and viscous forces and linearize

$$\frac{\partial\varrho}{\partial t} + \varrho_0\frac{\partial v}{\partial x} = 0$$
$$\frac{\partial v}{\partial t} + \frac{1}{\varrho_0}\frac{\partial p}{\partial x} = 0$$

The constitutive equation

$$\frac{\partial\rho}{\partial x} = -\kappa p$$

gives

$$\frac{\partial\rho}{\partial p} = \frac{\rho}{\partial p}\frac{\partial p}{\partial x}$$

Hence

$$\frac{\partial\varrho}{\partial t} + \varrho_0\frac{\partial v}{\partial x} = 0$$
$$\frac{\partial v}{\partial t} + \frac{1}{\kappa\varrho_0^2}\frac{\partial\varrho}{\partial x}$$

Eliminate ϱ

$$\frac{\partial^2 v}{\partial x^2} - \frac{1}{\kappa\varrho_0}\frac{\partial^2 v}{\partial t^2} = 0$$

Wave equation. Propagation velocity.

Water Hammer without Friction

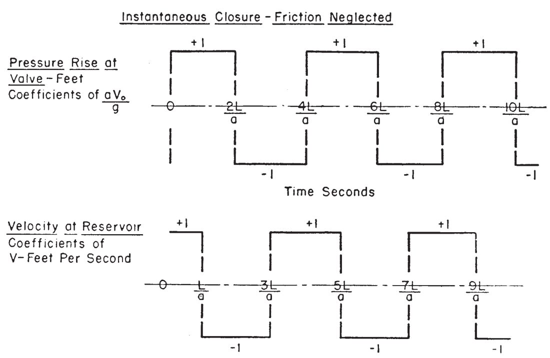


Fig. 1-3. Simple conduit—instantaneous closure—friction neglected.

Water Hammer with Friction

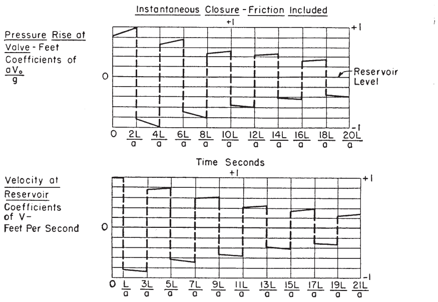
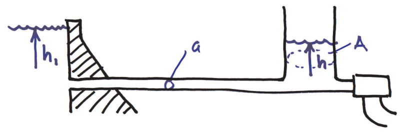


FIG. 1-4. Simple conduit—instantaneous closure—friction included.

Surge Tanks



- Use surge tank to avoid pressure transients due to water hammer
- Instabilities have been observed
- How to avoid them

Modeling

Mass balance

$$A\frac{dh}{dt} = av - a_u v_u$$

Energy balance. Follow a stream-line along the tube

$$\ell\frac{dv}{dt} + \frac{c}{2}v^2 + gh - gh_1 = 0$$
$$v_u^2 = 2gh$$
$$P = \frac{1}{2}\varrho a_u v_u^3$$

The model

$$\frac{dh}{dt} = -\frac{a_u}{A}\sqrt{2gh} + \frac{a}{A}v$$
$$\frac{dv}{dt} = \frac{g}{\ell}(h_1 - h) - \frac{c}{2\ell}v^2$$
$$P = \frac{1}{2}\varrho a_u v_u^3 = \frac{1}{2}\varrho a_u (2gh)^{3/2}$$

Replace v^2 by $v|v|$ to allow for bidirectional flow!

Linearization

Steady state solution

$$h = \frac{a^2}{a^2 + ca_u^2}h_1$$
$$v = \sqrt{\frac{a_u^2}{a^2 + ca^2}}\sqrt{2gh_1}$$

Linearization

$$\frac{d\delta h}{dt} = -\frac{a_u\sqrt{2gh}}{A}\frac{\delta h}{h} + \frac{a}{A}\delta v$$
$$\frac{d\delta v}{dt} = -\frac{g}{\ell}\delta h - \frac{(1+c)v}{\ell}\delta v + \frac{g}{\ell}\delta h_1$$

Linearization

$$\frac{dh_2}{dt} = -\frac{a_3}{A_2}\sqrt{2gh_2} + \frac{a_1}{A_2}v$$
$$\frac{dv}{dt} = \frac{g}{\ell}(h_1 - h) - \frac{c}{2\ell}v^2$$
$$P = \frac{1}{2}\varrho A_2 (2gh)^{3/2}$$

Thoma's Formula

Introduce a control system that changes a_u to keep output power constant, i.e. $P_0 = \frac{\varrho}{2}a_u v_u^3$

Hence

$$a_u v_u = \frac{2P_0}{\rho v_u^2} = \frac{P_0}{\rho gh}$$

The system is then described by

$$\frac{dh}{dt} = -\frac{P_0}{Ag\varrho h} + \frac{a}{A}v$$
$$\frac{dv}{dt} = \frac{g}{\ell}(h_1 - h) - \frac{c}{2\ell}v^2$$

$$\frac{d\delta h}{dt} = \frac{P_0}{Ag\varrho h^2}\delta h + \frac{a}{A}\delta v$$
$$\frac{d\delta v}{dt} = -\frac{g}{\ell}\delta h - \frac{cv}{\ell}\delta v + \frac{g}{\ell}\delta h_1$$

We have $\frac{P_0}{Ag\varrho h^2} = \frac{av}{Ah}$. Hence

$$\frac{d\delta h}{dt} = \frac{av}{Ah}\delta h + \frac{a}{A}\delta v$$
$$\frac{d\delta v}{dt} = -\frac{g}{\ell}\delta h - \frac{cv}{\ell}\delta v + \frac{g}{\ell}\delta h_1$$

Thoma's Formula, Cont

The linearized equation

$$\frac{d\delta h}{dt} = \frac{av}{Ah}\delta h + \frac{a}{A}\delta v$$
$$\frac{d\delta v}{dt} = -\frac{g}{\ell}\delta h - \frac{cv}{\ell}\delta v + \frac{g}{\ell}\delta h_1$$

Stability conditions

$$\frac{cv}{\ell} > \frac{av}{Ah}$$
$$\frac{ag}{A\ell} > \frac{acv^2}{Ah\ell}$$

Hence $Ach > a\ell$ and $\frac{cv^2}{gh} < 1$.

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Many Examples

- Incompressible
  - Chains of Dams
  - Hydroelectric Power Stations
  - Sloshing in rockets and milk packages
- Compressible
  - Pressure regulators
  - Pneumatic Controllers
  - Gas distribution networks
- Mixed systems
  - Active damping
  - Vibration isolation
  - Head Boxes for Paper Machines

Conclusions

- An area where physics is difficult and behavior rich
- Essential to understand the fundamentals
- Good demonstrations of balance equations and constitutive equations
- System theory and physics
- Learn differential geometry instead of vector calculus
- Many examples
- Good libraries missing