Automotive Modeling—An Overview of Model Components

Contents:

Lecture on May 5: Mathias Strandberg from Modelon will discuss automotive modeling using *Modelica and Modelon Impact*.

Automotive Modeling—An Overview of Model Components

Contents:

Introduction

- A large community.
- Self-driving cars accelerate development and extend needs and scope for automotive modeling.
- Large user groups for Modelica.
- Vehicular systems lend themselves well to component-based DAE modeling.
- This lecture aims to provide an overview of some model components common for automotive modeling.
- Many opportunities for course projects in this area.

Model Components for a Vehicle

A vehicle model typically consists of more or less complex models of the following components:

- Powertrain and braking systems,
- Wheels and tire dynamics,
- Steering and suspension dynamics,
- Chassis dynamics.

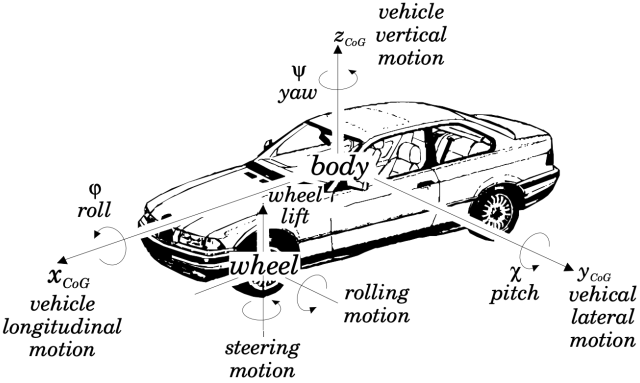
In addition: Driver and environment modeling important for automotive simulations.

Models for Different Purposes

- What is the purpose of the vehicle model?
- Wide range of applications for vehicle models, e.g.,
  - simulation (vehicle design, validation, control design),
  - dynamic optimization,
  - code generation for embedded real-time execution.
- What level of fidelity is required to capture essential dynamics?
- Models for dynamic optimization imply certain considerations (e.g., differentiation).

Vehicle Coordinate Frames

An illustration of involved vehicle coordinate frames and variables from [Kiencke & Nielsen, 2005]:



Automotive Modeling—An Overview of Model Components

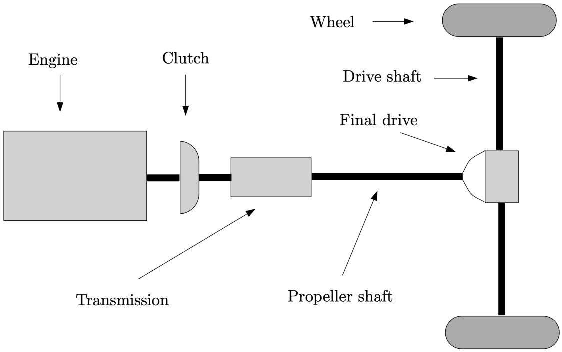
Contents:

Propulsion System for a Vehicle

- Drivetrain includes all components required to deliver power to the driving wheels of the vehicle from the engine/motor.
- Powertrain includes drivetrain and engine/motor.
  - Internal combustion engines (diesel, gasoline, ethanol, etc.).
  - Battery-electric vehicles with electric motor.
  - Hybrid vehicles (internal combustion engine and electric motor).
  - Fuel-cell electric vehicles.
- Dedicated drive cycles (driving missions) for system and control design.

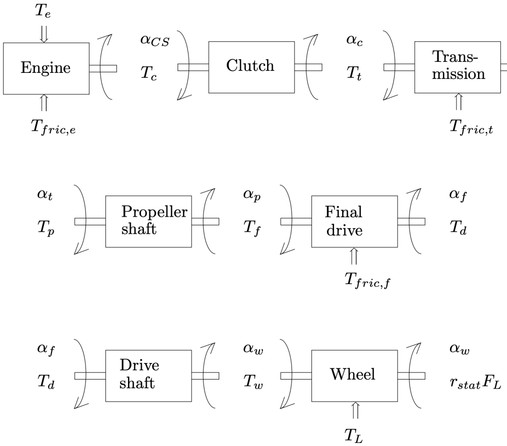
Vehicle Powertrain Model

- Engine/motor, clutch, transmission (gear box), shafts, and wheels.
- Often non-linear flexibilities in clutch and shafts.
- Illustration of a powertrain model from [Kiencke & Nielsen, 2005] for a rear-wheel driven vehicle:



Powertrain Model

- Rotational angles and torques involved in a powertrain model from [Kiencke & Nielsen, 2005]:



- F_{L} is the resulting traction force on the wheel moving the vehicle forward.

Aerodynamic and Rolling Resistance

- A low-complexity model for aerodynamic resistance, or air drag (v speed, ρ_{air} air density) [Kiencke & Nielsen, 2005]:

  $$F_{\mathrm{wind}} = \frac{1}{2} c_{\mathrm{air}} A_L \rho_{\mathrm{air}} v^2$$

  with c_{air} drag coefficient and A_L vehicle cross-section area.
- Relation based on fluid dynamics (Lord Rayleigh).
- Tabulated values for different vehicles. Average values of drag area $c_{\mathrm{air}} A_L$ for a passenger car are 0.5–2.5 m^2.
- Complex models for aerodynamic resistance (*cf.* racing cars).
- A low-complexity model for rolling resistance (m mass):

  $$F_{\mathrm{R}} = m(c_1 + c_2 v)$$

  where coefficients c_1 and c_2 depend on, e.g., tire properties.

Propulsion Forces

- Gravity force contributes with $-mg\sin(\chi_{\mathrm{road}})$ for a road with angle χ_{road}.
- Summing the forces involved in the vehicle propulsion in the longitudinal direction gives using Newton's second law of motion:

$$m\dot{v} = \underbrace{F_{\mathrm{L}}}_{\text{Traction force}} \underbrace{-F_{\mathrm{wind}} - F_{\mathrm{R}} - mg\sin(\chi_{\mathrm{road}})}_{\text{Opposing forces}}$$

where F_{L} is the traction force from the wheels delivered by the powertrain system.

Automotive Modeling—An Overview of Model Components

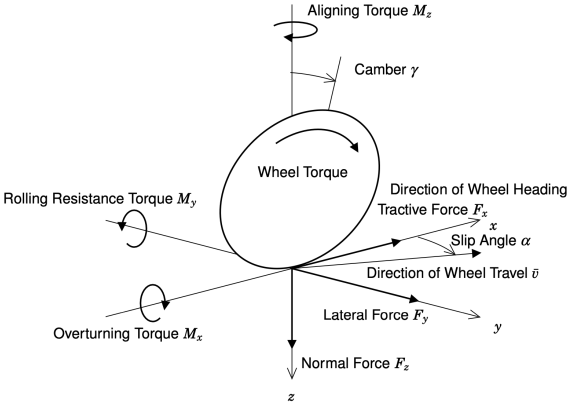
Contents:

Braking Systems for a Vehicle

- Brakes for reducing speed of the vehicle, often friction-based with mechanical device.
  - Disc brake or drum brake.
- Regenerative braking by electric motors (energy recovery by converting the kinetic energy).
- Many control systems related to braking:
  - Anti-lock braking system (ABS), maintain traction by avoiding wheel lock during braking.
  - Yaw control and Electronic Stability Control (ESC), individual-wheel braking.
  - Autonomous emergency braking systems.

Wheel Dynamics

- Forces and torques involved on a wheel, illustration from Ph.D. Thesis [Svendenius, 2007] based on SAE convention.



Wheel Slips

- Longitudinal slip ratio κ and lateral slip angle α for the wheel.
- Let R_w be the wheel radius, ω_i the angular velocity, and $v_{x,i}$, $v_{y,i}$ the longitudinal and lateral velocities for wheel i.
- Longitudinal slip ratio [Pacejka, 2006]:

$$\kappa_i = \frac{R_w \omega_i - v_{x,i}}{v_{x,i}}, \; i \in \{f, r\} \text{ or } \{1, 2, 3, 4\}$$

- Lateral slip angle with relaxation length [Pacejka, 2006]:

$$\dot{\alpha}_i \frac{\sigma}{v_{x,i}} + \alpha_i = -\arctan\left(\frac{v_{y,i}}{v_{x,i}}\right), \; i \in \{f, r\} \text{ or } \{1, 2, 3, 4\}$$

  where σ is the relaxation length.
- Body slip $\beta = \arctan\left(\frac{v_y}{v_x}\right)$, where v_x, v_y are longitudinal and lateral velocities at vehicle center of mass.

Automotive Modeling—An Overview of Model Components

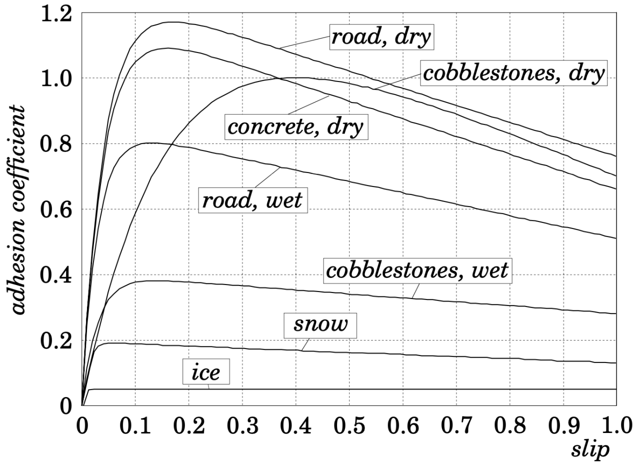
Contents:

Tire–Road Interaction Models

- Tire important part of the wheel.
- Friction between tire and road surface allows acceleration and deceleration of the vehicle as well as cornering (longitudinal F_x and lateral forces F_y).
- A vast plethora of models exist for modeling such dynamics.
- Dynamics depends on tire, road surface, temperature, normal load, etc. Thus, *tire–road interaction models*.

Longitudinal Tire Forces

Longitudinal tire forces as function of slip for different road surfaces, from [Kiencke & Nielsen, 2005].



Linear Tire Model

- A first, linear model of the longitudinal F_x and lateral F_y tire forces:

$$F_x = C_\kappa \kappa$$
$$F_y = C_\alpha \alpha$$

- C_κ and C_α are the tire longitudinal and cornering stiffness, respectively.

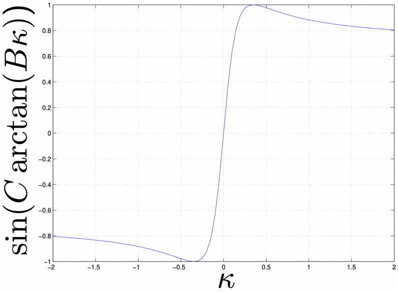
Tire–Road Interaction Models

- Pacejka's Magic Formula [Pacejka, 2006] slip-based model:

$$F_{x0} = \mu_x F_z \sin(C_x \arctan(B_x\kappa - E_x(B_x\kappa - \arctan(B_x\kappa))))$$

$$F_{y0} = \mu_y F_z \sin(C_y \arctan(B_y\alpha - E_y(B_y\alpha - \arctan(B_y\alpha))))$$

- Empirical model, calibrated based on experimental data.
- Hans B. Pacejka (TU Delft), book "Tyre and Vehicle Dynamics", co-founder of journal *Vehicle System Dynamics*.



Tire–Road Interaction Models: Combined Slip

- Friction ellipse for modeling of lateral forces (often with F_x as input):

$$F_y = F_{y0}\sqrt{1 - \left(\frac{F_x}{\mu_x F_z}\right)^2}$$

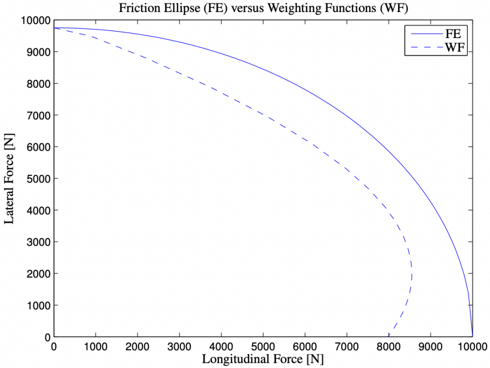
- Weighting functions for combined longitudinal and lateral tire forces [Pacejka, 2006]:

$$B_{x\alpha} = B_{x1}\cos(\arctan(B_{x2}\kappa)),\ \ G_{x\alpha} = \cos(C_{x\alpha}\arctan(B_{x\alpha}\alpha)),$$
$$F_x = F_{x0}G_{x\alpha}$$

- Corresponding weighting functions and parameters for the lateral force.

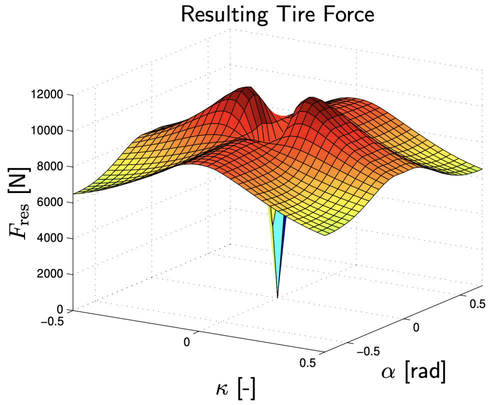
Friction Ellipse vs. Weighting Functions

- Comparison between Friction Ellipse and Weighting Functions from [Berntorp, 2013] for combined tire forces ($\alpha = 14$ deg.).
- Differences most prominent for low lateral tire forces.



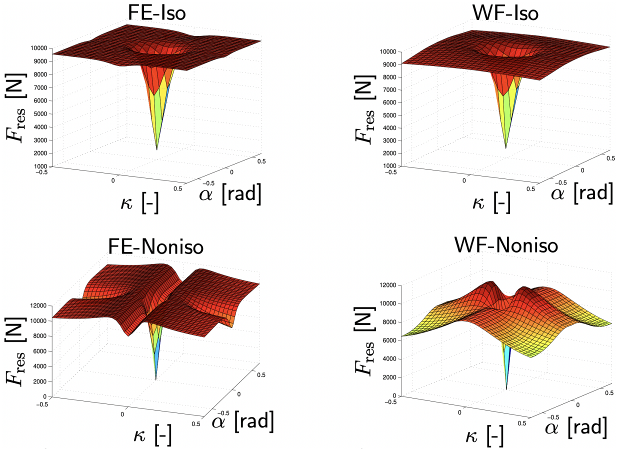
Empirical Pacejka Parameters and Weighting Functions

- Resulting tire-force surface from [Berntorp, Olofsson et al., 2013], with empirical parameters from [Pacejka, 2006].



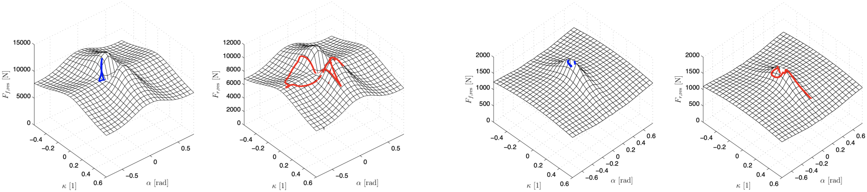
Tire–Road Interaction Model Calibration

- Tire-force surfaces from [Berntorp, Olofsson et al., 2013], with empirical parameters from [Pacejka, 2006] for friction ellipse (FE) and weighting functions (WF).



Force-Slip Diagrams

- Force-slip diagrams [Berntorp, Olofsson, et al, 2014] illustrate the normalized resultant tire-force F_{res}, as function of κ and α, where
$$F_{\text{res}} = \sqrt{F_x^2 + F_y^2}$$
- The trace from a vehicle maneuver is drawn on this surface.
- Gives valuable information about utilization of the tire–road friction potential.
- Examples for dry asphalt (left) and smooth ice (right) for a specific vehicle maneuver [Olofsson, Berntorp et al., 2013]:



Other Tire–Road Interaction Models

- The tire parameters can be scaled/modified to represent different road surfaces [Braghin et al., 2006].
- Models commonly used for simulation, not always sufficient for optimization.
- Friction is a complex phenomenon (recall previous lecture).
- Other common models are, e.g., Brush models, Dugoff / HSRI model, and Burckhardt model [Kiencke & Nielsen, 2005].
- Also transient friction models like LuGre and SWIFT have been proposed and adapted to tire modeling [Pacejka, 2006; Kiencke & Nielsen, 2005; Svendenius, 2007].

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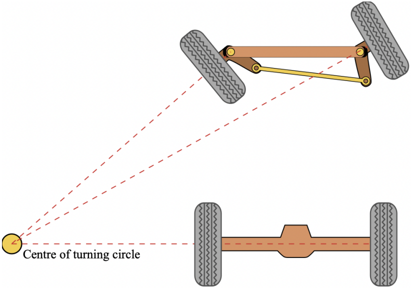
Contents:

Steering Dynamics

- The steering ratio: ratio between turning the steering wheel and the actual rotation of the wheels.
  - Usually in the range of 10–20:1 for passenger cars.
- The camber angle: angle between vertical axis of wheel and vertical axis of vehicle, with perspective from the front.
  - Affects the handling dynamics of the car in interaction with the suspension system, often utilized in racing.

Steering Kinematics—Ackermann Turning

- Left and right wheels on the axle moving with different curve radii.
- Steering mechanism known as Ackermann steering geometry (horse carriages, Georg Lankensperger, Rudolph Ackermann, 1817-1818).
- Quasi-static considerations for determining steering kinematics. Implications on vehicle dynamics.



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Suspension Dynamics

- Often used concepts:
  - Unsprung mass: Wheels and the suspension system.
  - Sprung mass: Carried by the suspension system (vehicle body).
- A first, linear model for a four-wheeled vehicle would be rotational inertia-spring-damper systems for roll and pitch directions.
- Moment τ_ϕ produced by the suspension system in the roll direction modeled by

$$\tau_\phi = (K_{\phi,f} + K_{\phi,r})\phi + (D_{\phi,f} + D_{\phi,r})\dot{\phi}$$

- Moment τ_θ in the pitch direction modeled according to

$$\tau_\theta = K_\theta \theta + D_\theta \dot{\theta}$$

where K and D are stiffness and damping parameters.

Dynamic Equations for Load Transfer

- A suspension system implies load transfer when accelerating/decelerating (i.e., time-varying normal forces).
- Dynamic equations for longitudinal load transfer:

$$(F_{z,1} + F_{z,2})l_f - (F_{z,3} + F_{z,4})l_r = K_\theta \theta + D_\theta \dot{\theta}, \quad \sum_{i=1}^{4} F_{z,i} = mg$$

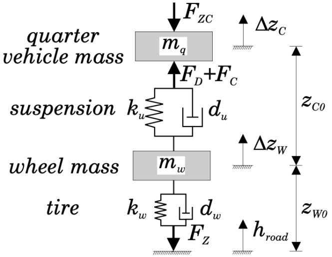
  where $F_{z,i}$, $i \in \{1, 2, 3, 4\}$, are the normal forces on each wheel and l_f, l_r are front and rear distances from the wheel axle to center of mass.
- Lateral load transfer:

$$\begin{aligned} -w(F_{z,1} - F_{z,2}) &= K_{\phi,f}\phi + D_{\phi,f}\dot{\phi}, \\ -w(F_{z,3} - F_{z,4}) &= K_{\phi,r}\phi + D_{\phi,r}\dot{\phi} \end{aligned}$$

  where w is half of the vehicle track width.

Quarter Model for Suspension Dynamics

- The quarter model is common for modeling and designing suspension systems (including control for active damping).
- One version from [Kiencke & Nielsen, 2005] illustrated in the figure.



m_q	: *quarter vehicle mass*
m_w	: *wheel mass*
k_u, k_w	: *spring constants*
d_u, d_w	: *damping constants*
z_C	: *height of contact point between chassis and undercarrige*
z_{C0}	: *static spring displacement*
z_W	: *height of contact point between wheel and road*
h_{road}	: *road height*
F_Z	: *wheel ground contact force*
F_{ZC}	: *vertical chassis force*
F_C	: *spring force*
F_D	: *damping force*

Automotive Modeling—An Overview of Model Components

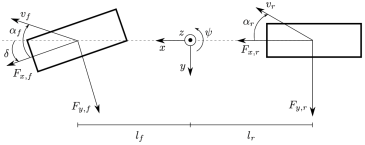
Contents:

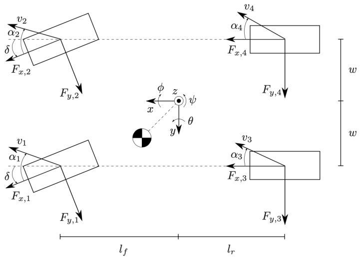
Characteristics of Chassis Dynamics

- Model equations for the chassis dynamics derived from analytical mechanics.
- Newton-Euler or Euler-Lagrange approach (recall previous lectures) to establish a differential-algebraic equation (DAE) system.
- Principles are straightforward, but often a time-consuming and slightly tedious process to derive the equations.
    - Tools for symbolic manipulation of variables useful.
- Extensive libraries with models of varying fidelity exist.

Chassis Dynamic Models

- Varying complexity of chassis models possible.
- Examples for a four-wheel vehicle include the simplified single-track model (**upper figure**) and the double-track model (**lower figure**).
- Double-track model:
  - Roll and pitch dynamics and associated load transfer.
  - Control inputs: Steering angle δ and wheel torques T_1, T_2, T_3, and T_4.





Single-Track Vehicle Model

- Single-track model equations [Berntorp, Olofsson et al., 2014]:

$$
\begin{aligned}
\dot{v}_x - v_y\dot{\psi} &= \frac{1}{m}(F_{x,f}\cos(\delta) + F_{x,r} - F_{y,f}\sin(\delta)) = \frac{F_X}{m}, \\
\dot{v}_y + v_x\dot{\psi} &= \frac{1}{m}(F_{y,f}\cos(\delta) + F_{y,r} + F_{x,f}\sin(\delta)) = \frac{F_Y}{m}, \\
I_{zz}\ddot{\psi} &= l_f F_{y,f}\cos(\delta) - l_r F_{y,r} + l_f F_{x,f}\sin(\delta) = M_Z,
\end{aligned}
$$

- F_X, F_Y, and M_Z are the global forces, δ is the steering angle, and I_{zz} is the inertia about the z-axis.
- Longitudinal tire forces F_x (or wheel torques) and steering angle δ as inputs.

Double-Track Model—Translational Motion

- Equations for a double-track vehicle model more extensive, see [Berntorp, 2013] for a full derivation.
- The model equations for translation motion along x and y are:

$$
\begin{aligned}
\dot{v}_x - v_y\dot{\psi} = h\Big(& \sin(\theta)\cos(\phi)(\dot{\psi}^2 + \dot{\phi}^2 + \dot{\theta}^2) - \sin(\phi)\ddot{\psi} - 2\cos(\phi)\dot{\phi}\dot{\psi} \\
& - \cos(\theta)\cos(\phi)\ddot{\theta} + 2\cos(\theta)\sin(\phi)\dot{\theta}\dot{\phi} \\
& + \sin(\theta)\sin(\phi)\ddot{\phi}\Big) + \frac{F_X}{m} \\
\dot{v}_y + v_x\dot{\psi} = h\Big(& -\sin(\theta)\cos(\phi)\ddot{\psi} - \sin(\phi)\dot{\psi}^2 - 2\cos(\theta)\cos(\phi)\dot{\theta}\dot{\psi} \\
& + \sin(\theta)\sin(\phi)\dot{\phi}\dot{\psi} - \sin(\phi)\dot{\phi}^2 + \cos(\phi)\ddot{\phi}\Big) + \frac{F_Y}{m},
\end{aligned}
$$

Double-Track Model—Global Forces

- The global forces for the translational motion are:

$$
\begin{aligned}
F_X &= F_{x,1}\cos(\delta) - F_{y,1}\sin(\delta) + F_{x,2}\cos(\delta) - F_{y,2}\sin(\delta) \\
&\quad + F_{x,3} + F_{x,4}, \\
F_Y &= F_{x,1}\sin(\delta) + F_{y,1}\cos(\delta) + F_{x,2}\sin(\delta) + F_{y,2}\cos(\delta) \\
&\quad + F_{y,3} + F_{y,4}
\end{aligned}
$$

Double-Track Model—Yaw Dynamics

- Let I_{xx}, I_{yy}, and I_{zz} be the inertia for the respective direction.
- The dynamic equation for ψ (yaw motion) is given by:

$$\ddot{\psi}(I_{xx}\sin(\theta)^2 + \cos(\theta)^2(I_{yy}\sin(\phi)^2 + I_{zz}\cos(\phi)^2))$$
$$= M_Z - h\Big(F_X\sin(\phi) + F_Y\sin(\theta)\cos(\phi)\Big),$$

where the global moment is:

$$M_Z = l_f\Big(F_{x,1}\sin(\delta) + F_{x,2}\sin(\delta) + F_{y,1}\cos(\delta) + F_{y,2}\cos(\delta)\Big)$$
$$+ w_f\Big(-F_{x,1}\cos(\delta) + F_{x,2}\cos(\delta) + F_{y,1}\sin(\delta) - F_{y,2}\sin(\delta)\Big)$$
$$- l_r(F_{y,3} + F_{y,4}) - w_r(F_{x,3} + F_{x,4}).$$

Double-Track Model—Pitch Dynamics

- The dynamic equation for θ (pitch motion) is given by:

$$
\begin{aligned}
\ddot{\theta}(I_{yy}\cos(\phi)^2 + I_{zz}\sin(\phi)^2) &= -K_\theta\theta - D_\theta\dot{\theta} \\
&\quad + h\Big(mg\sin(\theta)\cos(\phi) - F_X\cos(\theta)\cos(\phi)\Big) \\
&\quad + \dot{\psi}\Big(\dot{\psi}\sin(\theta)\cos(\theta)\big(\Delta I_{xy} \\
&\qquad + \cos(\phi)^2\Delta I_{yz}\big) - \dot{\phi}(\cos(\theta)^2 I_{xx} + \sin(\phi)^2\sin(\theta)^2 I_{yy} \\
&\qquad + \sin(\theta)^2\cos(\phi)^2 I_{zz}) - \dot{\theta}\big(\sin(\theta)\sin(\phi)\cos(\phi)\Delta I_{yz}\big)\Big)
\end{aligned}
$$

where $\Delta I_{xy} = I_{xx} - I_{yy}$ and $\Delta I_{yz} = I_{yy} - I_{zz}$.

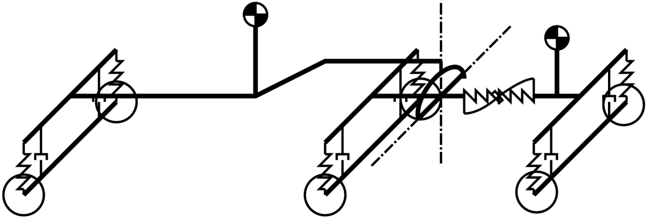
Double-Track Model—Roll Dynamics

- The dynamic equation for ϕ (roll motion) is given by:

$$
\begin{aligned}
&\ddot{\phi}(I_{xx}\cos(\theta)^2 + I_{yy}\sin(\theta)^2\sin(\phi)^2 + I_{zz}\sin(\theta)^2\cos(\phi)^2) \\
&\quad = -K_\phi \phi - D_\phi \dot{\phi} + h(F_Y \cos(\phi)\cos(\theta) + mg\sin(\phi)) \\
&\qquad + \dot{\psi}\Delta I_{yz}\left(\dot{\psi}\sin(\phi)\cos(\phi)\cos(\theta) + \dot{\phi}\sin(\theta)\sin(\phi)\cos(\phi)\right) \\
&\qquad + \dot{\psi}\dot{\theta}(\cos(\phi)^2 I_{yy} + \sin(\phi)^2 I_{zz}).
\end{aligned}
$$

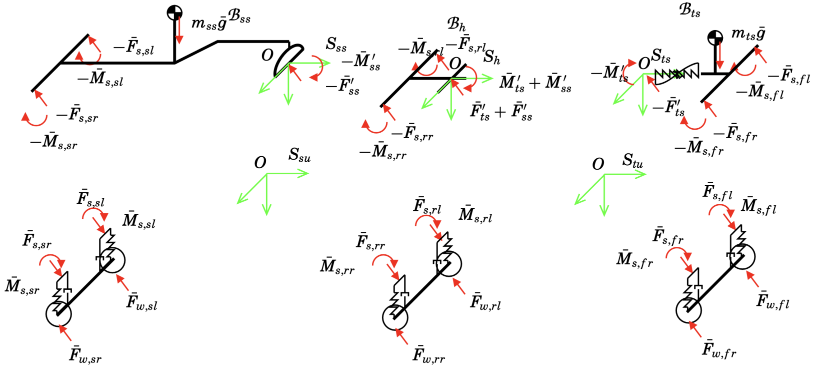
Tractor-Semitrailer Combinations (1/2)

- Corresponding models can also be derived for vehicles with additional degrees-of-freedom.
- Example in the figure: 9-DoF model from [Gäfvert & Lindgärde, 2001] for a tractor-semitrailer truck.
- Extensive model equations, beneficial with computer manipulation.



Tractor-Semitrailer Combinations (2/2)

- Principles for modeling are the same, though with additional involved coordinate frames.
- Illustration of tractor-semitrailer model with free-body diagram from [Gäfvert & Lindgärde, 2001]:

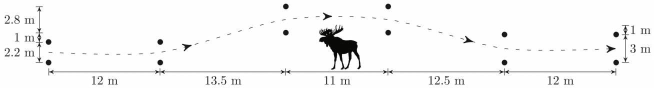


Automotive Modeling—An Overview of Model Components

Contents:

Experiments for Model Calibration

- Powertrain models identified, e.g., based on experiments on roads with different slopes, flexible modes included.
  - Measuring, e.g., engine/motor speed and torque, wheel and transmission speed for grey-box system identification.
- Dedicated test rigs for tire-force model calibration.
- Particular maneuvers for excitation of vehicle dynamics: e.g., double lane-change (ISO 3888-2:2011 test), fishhook, slalom maneuvers.
- Example of a double lane-change maneuver from [Anistratov, Olofsson et al., 2021]:



Measurement Setups for Tires and Vehicle Dynamics

- (Left) Mobile test rig for tire–road force measurements from [Svendenius, 2007], on the figure in Arjeplog for winter tests.
- (Right) Car equipped with sensor for vehicle-dynamics experiments (Linköping University).

Automotive Modeling—An Overview of Model Components

Contents:

Summary

- Automotive modeling covers many different model areas.
- Differential-algebraic equation systems natural for description of the model dynamics.
- Modelica offers a language for such model descriptions.
- Associated tools enable model simulations and dynamic optimization of DAEs.
- Mathias Strandberg from Modelon will describe how automotive modeling and simulation can be done using Modelica in the tool *Modelon Impact* during the lecture on May 5.

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