NORBERT WIENER PRIZE ACCEPTANCE SPEECH

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I am deeply honored to get this award, honored as a mathematician and also honored to have an award with Norbert Wiener's name attached. I had the great good fortune to know Wiener reasonably well for a few years before his death. I had the utmost admiration for him as a mathematician and as a man. Consequently, I'm just very pleased by the award. I also want to thank Garrett Birkhoff for his very kind comments. I wish I could now say "Thank you" and sit down. But when I got the letter, it read that it is customary, nay mandatory, for the recipient of an award like this to make a few comments for ten or fifteen minutes and so I will give in; I will accede to this mandate. As a matter of fact, I'd like to make a few comments, a few remarks about what dynamic programming is, where it's been, and where it's going. First of all, I'd like to explain what it is. Dynamic programming is a mathematical theory of multistage decision processes. The fact is that it is <u>the</u> mathematical theory of multistage decision processes, but I try to train myself diplomatically never to use the word "the" in mathematics or in science. It's a very bad word and I think all kinds of discourse would profit immeasurably if people got rid of the word "the" and just said "a". It is <u>a</u> theory of multistage decision processes.

An interesting question is, "Where did the name, dynamic programming, come from?" Now, as Garrett mentioned, the theory was developed in the 1950's which weren't good years for mathematical research. Some of you may remember that we had a very interesting gentleman in Washington named Wilson. He was Secretary of Defense, and he actually had a pathological fear and hatred of the word, research. I'm not using the term lightly; I'm using it precisely. His face would suffuse, he would turn red, and he would get violent if people used the term, research, in his presence. You can imagine how he felt, then, about the term, mathematical. At the time, I was employed by the Rand Corporation and it was employed by the Air Force, and the Air Force had Wilson as its boss, essentially. Hence, I felt I had to do something to shield Wilson and the Air Force from the fact that I was really doing mathematics inside the Rand Corporation. What title, what name, could I choose? Well, in the first place, I was interested in planning, in decision-making, in thinking. But planning, as you know, is not a good word for various reasons. I decided therefore to use the word, "programming". I wanted to get across the idea that this was dynamic, this was multistage, this was time-varying - I thought, let's kill two birds with one stone. Let's take a word which has an absolutely precise meaning, namely dynamic, in the classical physical sense. It also has a very interesting property as an adjective, and that is it's impossible to use the word, dynamic, in the pejorative sense. Sometime, when you want to amuse yourself, try thinking of some combination which will possibly give it a pejorative meaning. It's impossible. Thus, I thought dynamic programming was a good name. It was something not even a congressman could object to. So I used it as an umbrella for my activities.

Now, as I say, it's a theory of multistage decision processes and it's pretty clear, as you look around the world of engineering and economics, that there are a large number of multistage decision processes. As a matter of fact, since we operate in real time, everything <u>has</u> to be a multistage process. Sometimes for mathematical convenience, we simplify - we assume a steady state. But in actuality, this is never the case. Sooner or later, we have to face the fact that we are living in a time-varying world. We are living in a world where we cannot make a decision once and for all. We have to make sequences of decisions. We have to face new problems in new environments as we go along.

In this way I found large numbers of interesting processes. But then I used my mathematical license. If you ask, "What is it that the mathematician offers to the world?", it's this: he offers an "as if" property. The mathematician doesn't have to treat an engineering process as a multistage decision process. He can treat any process that can be interpreted as a multistage decision process. And so it became very interesting to look at a large number of other processes, some of them well entrenched in classical mathematics, like the calculus of variations, and see if one couldn't interpret them profitably as multistage decision processes. One can do this for the calculus of variations and one can get new computational techniques and new analytic results, a very much more elementary and intuitive derivation of the whole theory.

This then was the first stage - looking at processes that existed and seeing if I could interpret them within a common conceptual framework. The second stage was to take these processes and convert the multistage decision aspects into the problem of solving functional equations. Now what is interesting about this is that one encounters many new intriguing types of functional equations. This is an interesting pursuit in itself. But now we'll use the "as if" property again. It turns out that large numbers of functional equations can be interpreted as if they come from multistage decision processes. This is important because it gives you a hold on the equation which you don't have if you look at the equation by itself.

The third stage was the question of, can one really obtain numerical answers to numerical questions. And, at first (I think one should admit this), I had the attitude that many young mathematicians used to have, and one a number of young mathematicians probably still have, that is, to use the computer is to admit their defeat. I sort of scorned the digital computer. Obviously, anybody could use the digital computer to get numerical results - nothing hard about that...until I started trying to do it. And I was sort of amazed at how difficult it really was. It became a tremendous game and a tremendous challenge, and if it is a challenge now, think of what it must have been ten years ago, fifteen years ago, with the kinds of primitive digital computers that one had then. Now the interesting thing about it is that it is as much of a challenge now as it ever was - as a matter of fact, even more - because as the computers get bigger, you get more ambitious. You try harder problems; you want to get more results.

I began to encounter what I call the "curse of dimensionality." I began to realize that the real barrier that confronts the mathematician is not so much the conceptual barrier, not so much the analytic barrier, but the plain old arithmetic barrier - the barrier that has always confronted all men in science. How does one really use formulas and theories to get numerical results? And this is still one of my major interests. It began to become very, very clear that this is not a problem which is going to be solved by brute force. This is going to require the most sophisticated, the most difficult mathematics about. And I think what's interesting about this is it means that if we want to solve this type of problem, we have to understand the structure of the process. We have to really be able to decompose the process into simpler components. This means that we need a very thorough knowledge of algebra and topology.

Hence I think what's rather interesting about the development of the computer is that it is the use of the computer which makes it absolutely mandatory to understand the topology and the algebra of a large number of processes. And what we can hope is that we will see more and more of this type of activity. There have been a number of attempts in this area, and I might say that many years ago there was a very interesting person named Kron who was one of the first to realize that one should use topological ideas in the study of large systems. I think anybody who has tried to read Kron will say that Norbert Wiener is a model of lucid exposition in comparison to Gabriel Kron.

But this question of high dimensionality, this question of large systems - the problem is, can we really handle large systems? This is a mathematical problem. This is a scientific problem. This is an engineering problem. Now, I'd like to say something more. I think that not only is it a problem of the type I just mentioned, I think it's a problem of survival, that we have to begin to understand our society is a contrast of interconnecting, interacting large systems and that so many of the difficulties that we see today are the difficulties - not of inherent theory, good theory, bad theory, not of conspiracy but just the difficulties due to large systems. I think it's beginning to be realized that our systems are falling apart. We don't know how to administer them. We don't know how to control them. And it isn't at all obvious that we can control a large system in such a way that one remains stable. It may very well be that there is a critical mass - that when a system gets too large - it just gets automatically unstable. The problems then we see in our medical systems, in our educational systems, in our legal systems, in our transportation systems, in our garbage collection systems, all the systems you can probably think of, these are problems of instability. It may very well be that these are inherent, and this I'd like to call to your attention as a mathematical problem of great interest and importance and relevance. I think that what's important about so much of this is it shows rather clearly that the mathematician now occupies a central position in society, not only intellectually, but as far as the very existence of society is concerned. I would like to call this to the attention of a large number of young mathematicians who are very worried about the problem of relevance. At first sight, it seems very difficult to be relevant as a mathematician. I would say it is quite the contrary. I would say society now needs mathematicians and mathematics more than ever.