Lund University

Department of
AUTOMATIC CONTROL

Control Engineering AK, FRTF05

Exam October 28, 2024, 2pm–7pm

Points calculation and grading

Solutions and answers to all tasks must be clearly motivated. The exam is worth a total of 50 points. The point allocation is marked for each task.

Grade 3: at least 24 points
4: at least 34 points
5: at least 44 points

Allowed aids

Mathematical tables (TEFYMA or equivalent), the department's formula collection in control theory, and non-programmable calculators.

Exam results

The results will be reported via Ladok. The solutions will be available via Canvas. The time and place for reviewing the exam will be announced via Canvas.

1. A system is given by $Y(s) = G(s)U(s)$ with

$$G(s) = \frac{2}{s+1} - \frac{3}{s+3}$$

a. Calculate the step response $y(t)$ of the system, (i.e., when the input signal is $u(t) = \theta(t)$). (2 p)

b. Calculate the system's poles and zeros and plot them in a singularity diagram. (2 p)

c. Provide a state-space representation

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

for the system. (2 p)

2. A less skilled engineer failed in designing a critical process step in a newly built jam factory. This resulted in an unstable process. The engineer's more competent colleague managed to derive a model of the process

$$G_p(s) = \frac{1}{s-3}$$

and is hopeful that the process can be stabilized with a PI controller $G_r(s) = K\left(1 + \frac{1}{sT_i}\right)$, in accordance with the block diagram in figure 2.

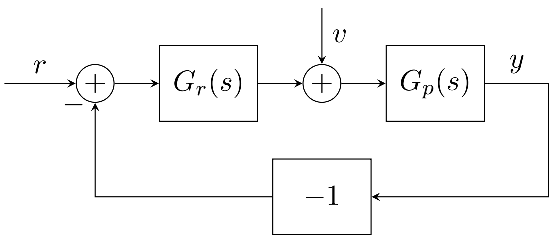

Figure 1 Block diagram for the process in problem 2.

a. Save the Nordic jam supply by determining all parameters $K > 0$ and $T_i > 0$ for which the PI controller stabilizes the process. (4 p)

b. Calculate the steady-state error when the PI-controlled process is subjected to a disturbance in the form of a ramp, $v(t) = t$. Assume the system setpoint is $r = 0$. (4 p)

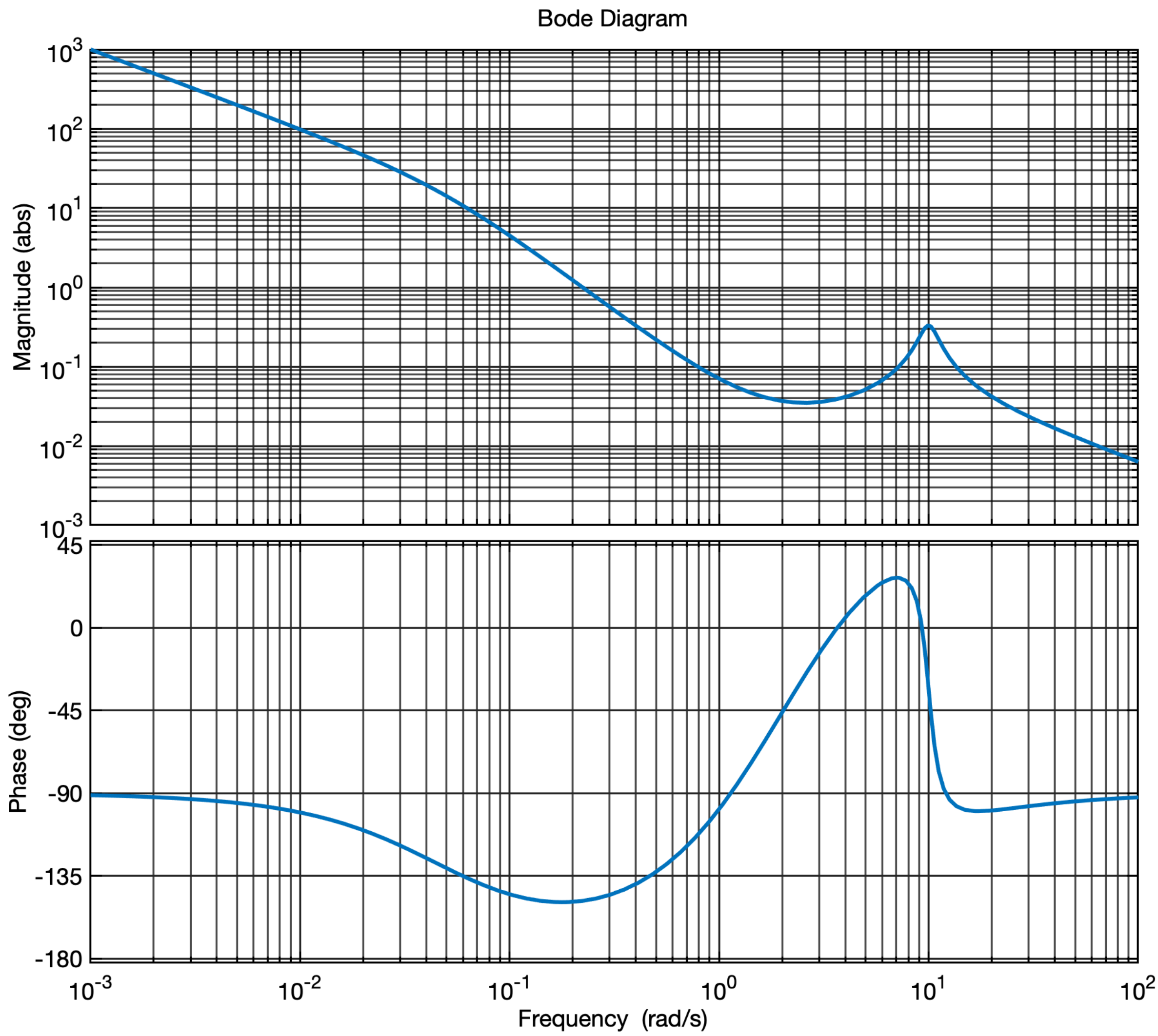

Figure 2 Bode diagram in Problem 3.

3. A servomotor has a Bode diagram as shown in Figure 2.

a. Use the Bode diagram to determine the output signal $y(t)$ after any transients have decayed, when the input is (2 p)

$$u(t) = \sin(0.1t).$$

b. What is the phase margin if the motor is feedback-controlled with a P-controller with gain $K = 0.1$? (2 p)

c. Explain why the Ziegler-Nichols method for tuning PID parameters will not work for a system with this Bode diagram. (2 p)

4. Consider the system

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u,$$
$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} x.$$

a. Is the system controllable ? (2 p)

b. Is the system observable ? (2 p)

c. Design a state feedback controller of the form (6 p)

$$u = -Kx + k_r r$$

The closed-loop system should have one pole at $s = -1$ and another at $s = -2$, and there should be no steady-state error.

5. Match the transfer functions G_1 to G_4 with the step responses A to F in Figure 3 (two figures are left over). Don't forget to justify your answer. (4 p)

$$G_1(s) = \frac{e^{-s}}{s+1}, \qquad G_2(s) = \frac{2-s}{s^2+2s+2},$$
$$G_3(s) = \frac{2}{s^2+4}, \qquad G_4(s) = \frac{1}{s^2+s+1}$$

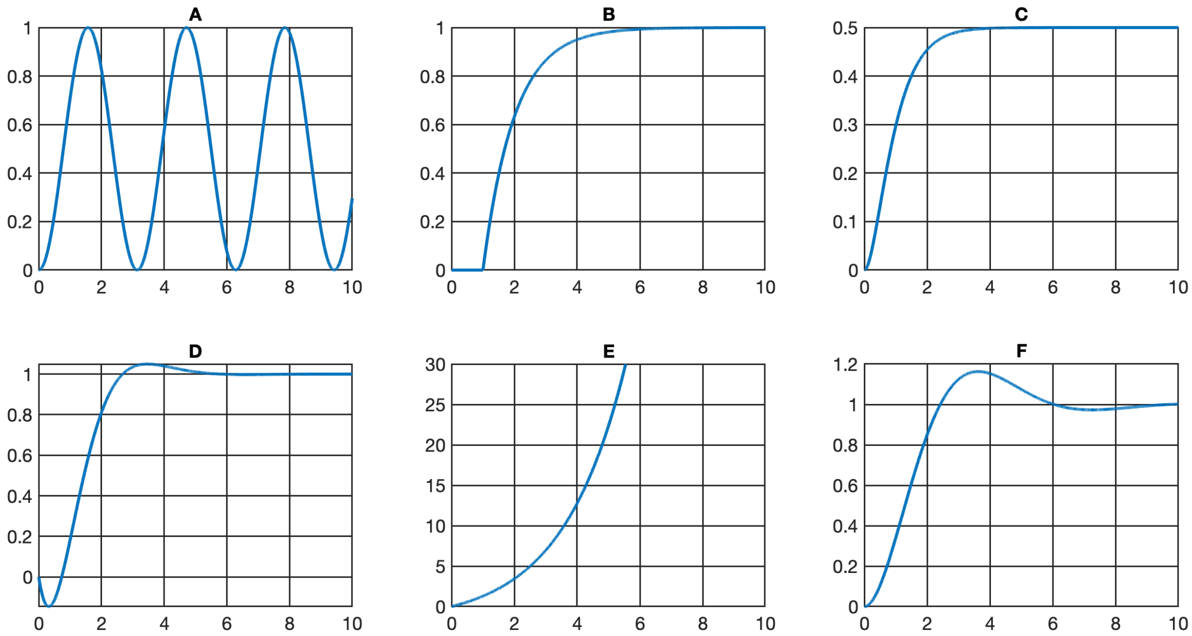

Figure 3 Step responses in Problem 5.

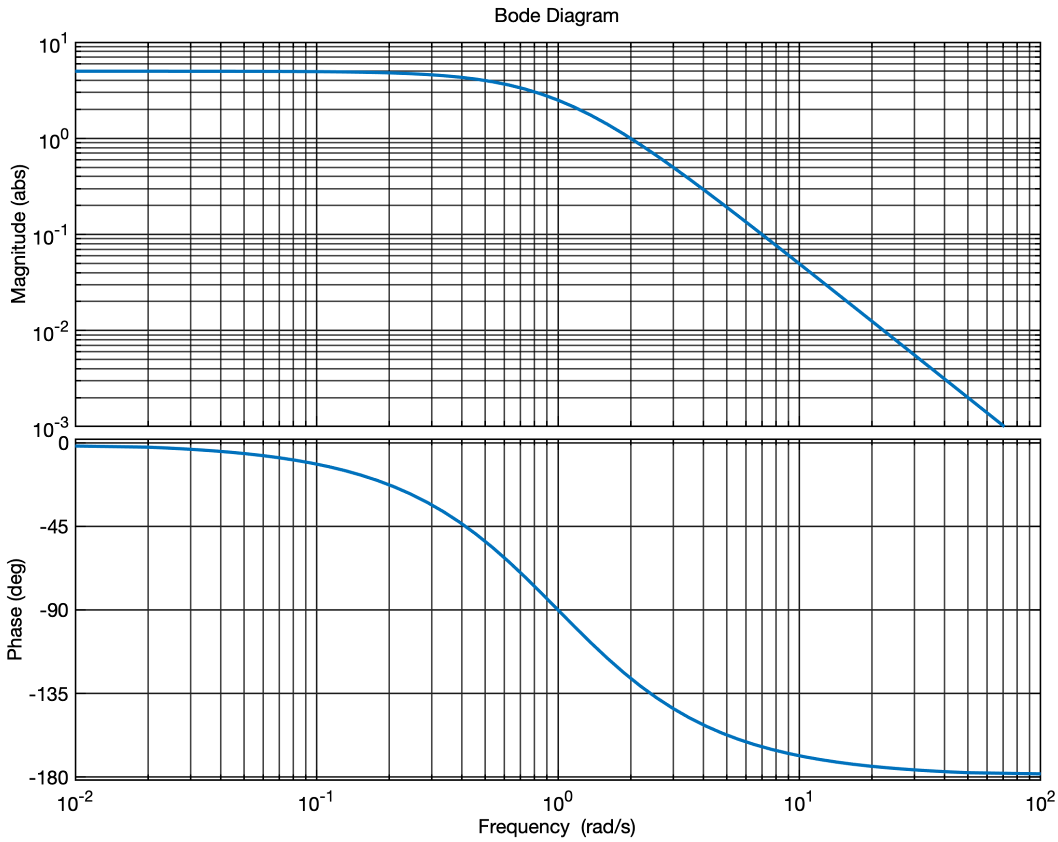

Figure 4 Bode diagram in Problem 6.

6. Engineering students Truls and Trula have a system whose Bode diagram is shown in Figure 4.

a. Trula wants to make the system faster and is considering adding a compensating link. Should she choose a lead or lag compensator? (1 p)

b. Trula would like the system to be 5 times faster. What should the new crossover frequency ω_c be for her new system? (1 p)

c. Truls reads in the lecture notes and thinks designing a compensating link is complicated. He claims that it is enough for Trula to increase the gain by a constant K to increase the speed. What K would be required to achieve the desired crossover frequency? (2 p)

d. What is the drawback of Truls' approach to solving the problem? (2 p)

e. The system in figure 4 has the transfer function

$$G_p(s) = \frac{b}{(s+a)^2}$$

Determine the parameters b and a from the Bode diagram. (2 p)

f. Help Trula design a compensating link that makes the system 5 times faster and gives a phase margin of 45 degrees. (4 p)

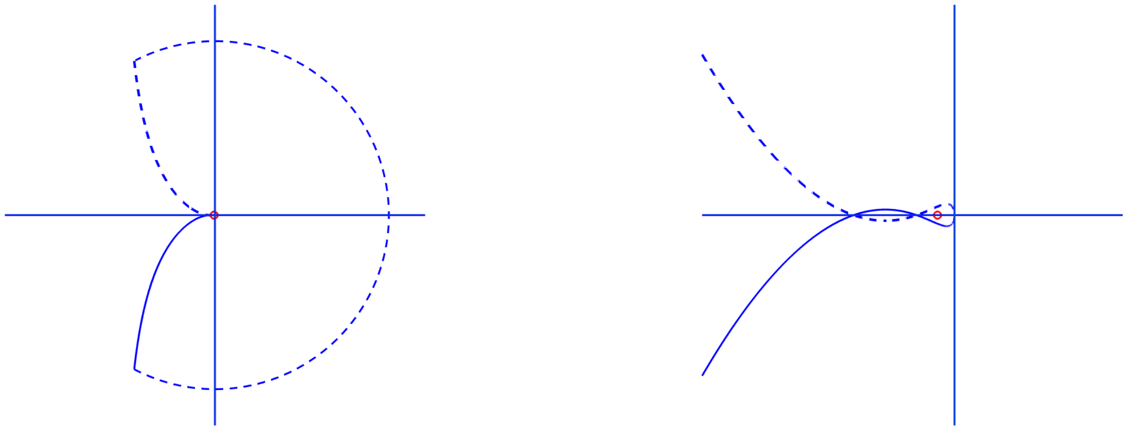

Figure 5 Nyquist plot in Problem 7.

7. In Figure 5, the (complete) Nyquist plot for the system

$$G_0(s) = \frac{(s+6)^2}{s(s+1)^2}$$

is sketched. A zoomed-in view is also provided to show what happens near the origin. The point -1 is marked with a small red circle.

a. You can see that the Nyquist plot intersects the negative real axis at two points. Calculate these points! (2 p)

Numerical hint: The polynomial $\omega^4 - 13\omega^2 + 36$ *has roots* $\omega = \pm 2$ *and* $\omega = \pm 3$.

If you could not solve the previous problem, you might assume in the next subproblem that the intersection points are -5 and -2.

b. The system G_0 is feedback-controlled with a P-controller, $u = -Ky$. Determine which gains $K > 0$ give an asymptotically stable closed-loop system. (2 p)