LUNDS UNIVERSITET

Institutionen för
**REGLERTEKNIK**

# Automatic Control , FRTF05

**Exam 14 March 2024 8–13**

### Scoring and Grading

Solutions and answers to all questions must be clearly justified. The exam totals 25 points. The scoring is marked for each question.

3: at least 12 points

4: at least 17 points

5: at least 22 points

### Permitted Materials

Mathematical tables (TEFYMA or equivalent), the department's formula collection in control engineering, and non-programmable calculators.

### Examination Results

Results will be reported via Ladok. The key will be available through Canvas. Time and place for exam review will be announced via Canvas.

**1.** A system is given by $Y(s) = G(s)U(s)$ with

$$G(s) = \frac{1}{s+1} + \frac{2}{s+3}$$

**a.** Calculate the system's poles and zeros and plot these in a singularity diagram. (1.5 p)

**b.** Is the system asymptotically stable, stable, or unstable? (0.5 p)

**c.** Calculate the system's step response $y(t)$, (i.e., when the input signal is $u(t) = \theta(t)$). (1 p)

**d.** Determine a differential equation in the form (specify the $a$ and $b$ coefficients)

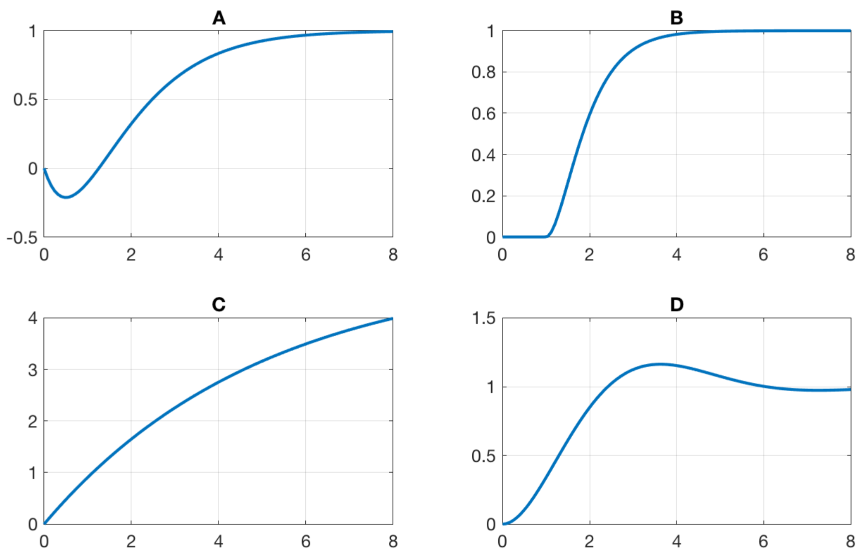
$$\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_1\dot{u}(t) + b_0 u(t)$$

that describes the system above for a general input signal $u(t)$. (1 p)

**2.** Match the step responses A to D in figure **??** with the transfer functions $G_1$ to $G_5$ (one will be left over). Do not forget to justify your answer. (2 p)

$$G_1(s) = \frac{1}{s+0.2}, \qquad G_2(s) = \frac{1-s}{(s+1)^2}, \qquad G_3(s) = \frac{4e^{-s}}{(s+2)^2}$$

$$G_4(s) = \frac{1}{s^2+s+1}, \qquad G_5(s) = \frac{1}{5s+1}$$




**Figur 1** Step responses in problem **??**.

**3.** Using a large electromagnet, one wishes to make a small permanent magnet levitate. The magnet's position in height, $y$, is modeled with the following nonlinear state model.

$$\begin{aligned}\dot{x}_1 &= -\frac{d}{m}x_1 + \frac{ku}{mx_2} - g \\ \dot{x}_2 &= x_1 \\ y &= x_2,\end{aligned}$$

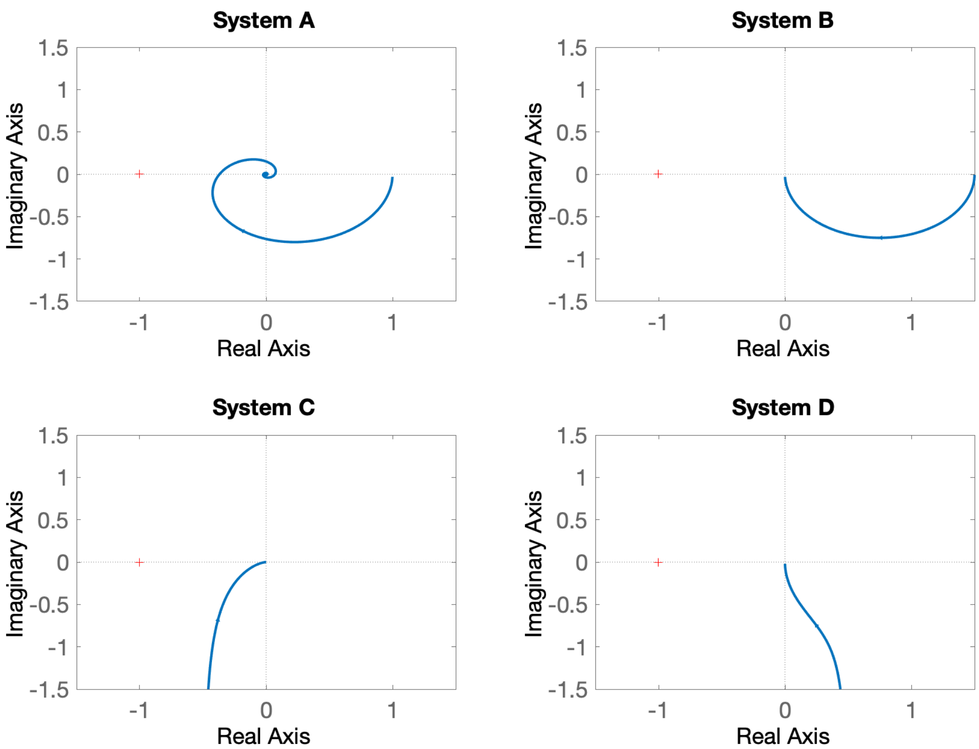
where $g$ is the gravitational acceleration, $u$ is the control current to the electromagnet, $m$ is the mass of the small magnet, and $k$ and $d$ are constants.

**a.** Show that $(x_1, x_2, u) = (0, \frac{k\alpha}{mg}, \alpha)$ is a stationary point ($\alpha > 0$ is arbitrary). (0.5 p)

**b.** Linearize the model around the stationary point above when $\alpha = g$. (1.5 p)

**4.** The Nyquist plots A-D for four different systems are shown in Figure **??**. The systems are described by the four transfer functions below. Match the plots A-D with the transfer functions 1-4. Justify your answers! (2 p)

$$G_1(s) = \frac{3}{s+2}, \quad G_2(s) = \frac{2}{s(s+2)}, \quad G_3(s) = \frac{e^{-s}}{(s+1)^2}, \quad G_4(s) = \frac{1+s}{s(1+s/2)}$$




**Figur 2** Nyquist diagrams A-D in problem **??**.

**5.** A process

$$G_P(s) = \frac{1-s}{s^2+2s+2}$$

is controlled with a proportional P-controller, i.e., $Y(s) = G_P(s)U(s)$ and $U(s) = KE(s) = K(R(s) - Y(s))$.

**a.** Calculate the closed-loop system poles. For which constants $K > 0$ is the closed-loop system asymptotically stable? (1 p)

**b.** Assume that we make a step change in the reference value. What can you say about the steady-state error, $\lim_{t\to\infty} e(t)$, for the cases when $K = 1$ and $K = 4$? (1 p)

**6.** The following system is to be controlled

$$\dot{x} = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

**a.** Show that the system is controllable. (1 p)

**b.** Design a state feedback $u = -Kx$ such that the poles are placed at $s = -2 \pm i$. (2 p)

**c.** Assume we have a sensor that measures $y = [1 \quad -1]\, x$. Is the system observable? (1 p)

**d.** If one constructs an observer/Kalman filter

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$

that estimates $x$ asymptotically correctly and gives observer dynamics with characteristic polynomial $p(s) = (s+2)(s+1)$, then the design equation yields that $L = 0$ (you do not need to verify this). Interpret this result! (1 p)

**7.** You want to control a process with the transfer function

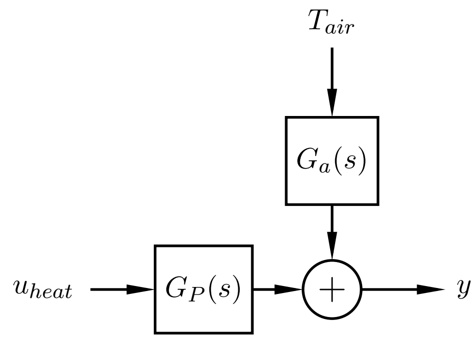
$$G_P(s) = \frac{10}{s^2}$$

so that the system has a crossover frequency $\omega_c = 10$ rad/s and a phase margin of 30 degrees. Design a suitable compensating link $G_R(s)$ that achieves this. Also sketch the resulting Bode plot (i.e., for $G_R(s)G_P(s)$). (3 p)

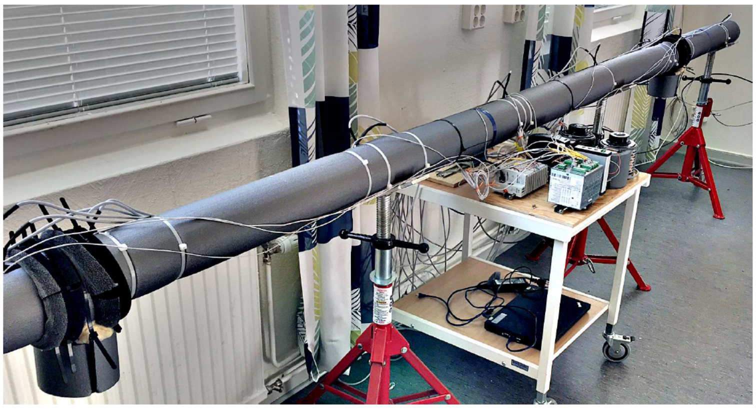
**8.** The so-called ”phase reference system” serves an important function at ESS. It consists of a roughly 500 meter long tube that distributes a radio signal responsible for the synchronization of all parts of the facility, with an accuracy at the picosecond level. Since even small temperature variations can disturb the accuracy, the tube must be carefully temperature controlled. The temperature regulation is managed by a control system (designed at our department) that controls a heating signal $u_{heat}$. The goal is to maintain the temperature $y$ at a constant reference temperature, say $r = 35°C$. The surrounding air temperature, $T_{air}$, can vary, which disturbs the system.

A simplified block diagram of the system is shown in Figure **??**. Through experiments on a prototype system, see figure **??**, the following dynamics have been determined

$$G_P(s) = \frac{K_p}{sT_p + 1}, \qquad G_a(s) = \frac{e^{-sL}}{(T_1 s + 1)(T_2 s + 1)}.$$




**Figur 3** The system in problem **??**. Here, $u_{heat}$ is the control signal and $T_{air}$ is the surrounding air temperature. The goal is to keep the output signal $y$ at a constant temperature $r$, despite variations in $T_{air}$.



**Figur 4** Prototype for the temperature control of the ESS phase reference system.

**a.** Determine formulas for $K$ and $T_i$ for a PI controller

$$u_{heat}(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right), \qquad e(t) = r(t) - y(t)$$

so that the closed-loop system has the characteristic polynomial $s^2+2\zeta\omega s+\omega^2$. ($K$ and $T_i$ will depend on $\zeta, \omega$ and the parameters in $G_P(s)$.) (2 p)

**b.** To reduce the impact of temperature variations in the surrounding air, one can measure $T_{air}$ and use feedforward control. Draw a block diagram where the control system with the PI controller from the previous task is complemented with a feedforward filter $G_{FF}(s)$. Also determine the filter $G_{FF}(s)$ that perfectly compensates for $T_{air}$. (2 p)

**c.** Upon a rapid decrease in the reference temperature $r$, it is noticed that $u_{heat}$ hits the lower limit $u_{heat,min} = 0$ that exists (the system can only heat, not cool). After the temperature $y$ has finally dropped to the new reference level, it is unfortunately noticed that $y$ continues to drop without the regulator reacting for a long time, and a large undershoot occurs before the regulator eventually starts heating again. What is the phenomenon called, and what modification to the regulator do you suggest? (1 p)