Lab in Dynamical systems and control TSRT21

Angle estimation using gyros and accelerometers

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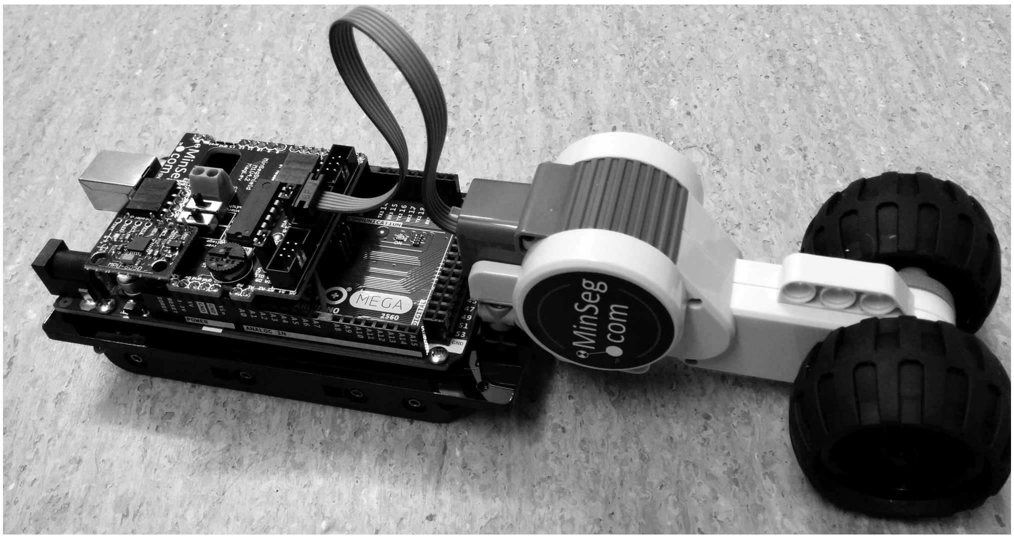

REGLERTEKNIK
AUTOMATIC CONTROL
LINKÖPING

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Chapter 1

Introduction

The purpose of this lab is to illustrate sensors, measurements, filtering and simple sensor fusion. Our goal is to estimate the tilt-angle of the MinSeg. By extracting information from both the accelerometers and the gyro and combining these using low-pass and high-pass filters, we will create a better estimate of the angle of the MinSeg, compared to a naive approach of using only the gyro, or only the accelerometer.

Figure 1.1: Intuitive Aerial AB in Linköping develops a camera-carrying Hexacopter. For high-precision autonomous flight (and filming), it is important to know the orientation (pitch-, roll- and yaw-angles) of the system. Gyros and accelerometers are used to extract as much information as possible, with as little error and noise as possible.

1.1 Hardware set-up

The lab is based on three main hardware components.

To begin with, we have a standard desktop computer. This computer is used to automatically develop and deploy code using MATLAB and SIMULINK models.

We use a board which is equipped with an accelerometer and a gyro. The board is built around an Arduino micro-controller which runs the auto-generated code. It also communicates with the desktop computer and thus allows us to look at the measurements.

The Arduino board together with the motor and wheels is called the MinSeg.

1.2 Trouble shooting

Complaints about COM port or connection when downloading to board Disconnect USB-cable and connect it again.

Chapter 2

Preparation

The questions below, and all questions throughout the document marked as **Preparation** must be done before attending the lab. Note that there are additional preparation exercises in Chapter 3.

Solutions to all questions should be available upon request from the lab assistant, and the preparation exercises in Chapter 3 are preferably written in this printed documented.

The scheduled time spent with the laboratory equipment is only a small part of the complete lab, as a major part is spent on the theoretical material during preparations.

Preparation 1 *Read Chapter 1 and 2 in the auxiliary course compendium.*

Preparation 2 *Gyros allows us to measure the angular velocity, which we can integrate (numerically in practice) to obtain the rotation angle. The problem is that the angular velocity measurements have errors.*

We typically divide errors into a fixed constant error (called bias or calibration error) and a faster varying error term (noise) which is 0 *on average. Consider the measurement of a true angular velocity* $\omega(t)$ *which we want to integrate to obtain the true rotation angle* $\theta(t) = \int_0^t \omega(\tau)d\tau$. *Let the measurement be* $\omega_m(t)$. *A simple description uses a bias* b, *noise amplitude* ϵ *and noise frequency* N,

$$\omega_m(t) = \omega(t) + b + \epsilon \sin(Nt) \tag{2.1}$$

Integrate this expression analytically and derive an expression for the estimated angle $\theta_m(t) = \int_0^t \omega_m(\tau)d\tau$ *(you assume initial conditions to be* 0*). The solution will consist of the true angle, a linear term, and a periodic term. Verify that the term in the error* $\theta(t) - \theta_m(t)$ *that depends on* b *will grow as time increases (called linear dift), but the term coming from the high-frequency noise will be limited and decrease the larger* N *is.*

Preparation 3 *To obtain the rotation angle* $\theta(t)$*, we have to compute the integral* $\theta(t) = \int_0^t \omega(\tau)d\tau$*. However, we cannot integrate perfectly on a computer with a finite number of samples, and must use an approximation. One such integral approximation is a rectangle approximation based on Euler backwards. With measurements* $\omega(kT_s)$ *for* $k = 0, 1, \ldots$ *and sample-time* T_s*, the approximation is given by*

$$\theta(kT_s) = \theta((k-1)T_s) + T_s\omega(kT_s) \tag{2.2}$$

Consider the case in Figure 2.1 where a signal $\omega(t)$ *has been sampled with* $T_s = 1$*. Illustrate graphically in the figure how the integral from* $t = 0$ *to* $t = 4$ *is computed, and confirm that the approximation leads to the estimate* $\theta(4) = 9.5$*. Note that the value* $\omega(0)$ *never is used in the backwards approximation. You initialize the angle estimate to* $\theta(0) = 0$*.*

Preparation 4 *When working with sampled signals, we often use the* z*-transform for notational convenience. Instead of working with* $\theta(kT_s)$ *we introduce its transform* $\boldsymbol{\theta}(z)$[1]*, and the unit delay-operator* $\frac{1}{z}$ *to express* $\theta((k-1)T_s)$*, i.e., the transform of* $\theta((k-1)T_s)$ *is* $\frac{1}{z}\boldsymbol{\theta}(z)$*. Introduce this notation for (2.2) and show that the integral computation (2.2) can be represented as*

$$\boldsymbol{\theta}(z) = \frac{T_s z}{z-1}\boldsymbol{\omega}(z) \tag{2.3}$$

Preparation 5 *Read the complete lab-pm. There are more theoretical questions in the pm which you are supposed to complete as preparation.*

[1] To avoid using capital greek letters, we indicate signal transforms by bold letters in this text

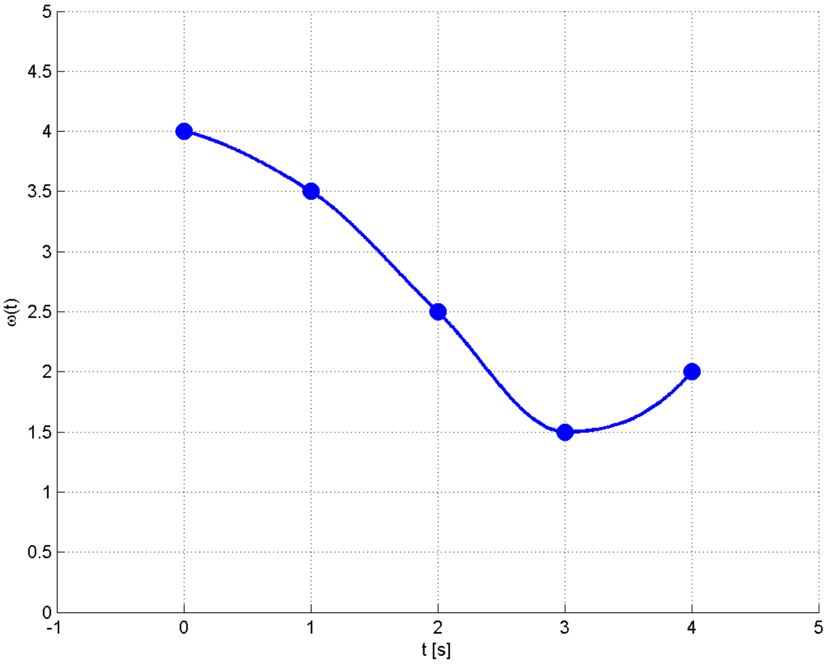

Figure 2.1: A continuous signal sampled (dots) with $T_s = 1s$.

Preparation 6 *Print this document. You must bring a physical copy to the lab.*

Chapter 3

The lab

The lab will primarily consist of experimentation and data collection, using theoretical results and strategies derived during your preparation.

Items labeled **Preparation** are questions you are supposed to solve and fill out before attending the lab.

Items labeled **Task** are questions you answer and solve when attending the lab and have access to the hardware.

3.1 Gyro and accelerometer

The board is equipped with a gyro measuring angular velocity around 3 axes, and an accelerometer measuring linear acceleration in the same axes. The accelerometer and gyro hardware are placed on the blue board as indicated in Figure 3.1. The measurements are done in a body-fixed coordinate system as indicated in Figure 3.2.

The gyro

Although the gyro is the natural sensor to use for estimating an angle (as it measures the derivative of an angle), it has some problems. To begin with, the gyro measurements will only give us angles relative to initial orientation, as we simply integrate the angular velocity. In many

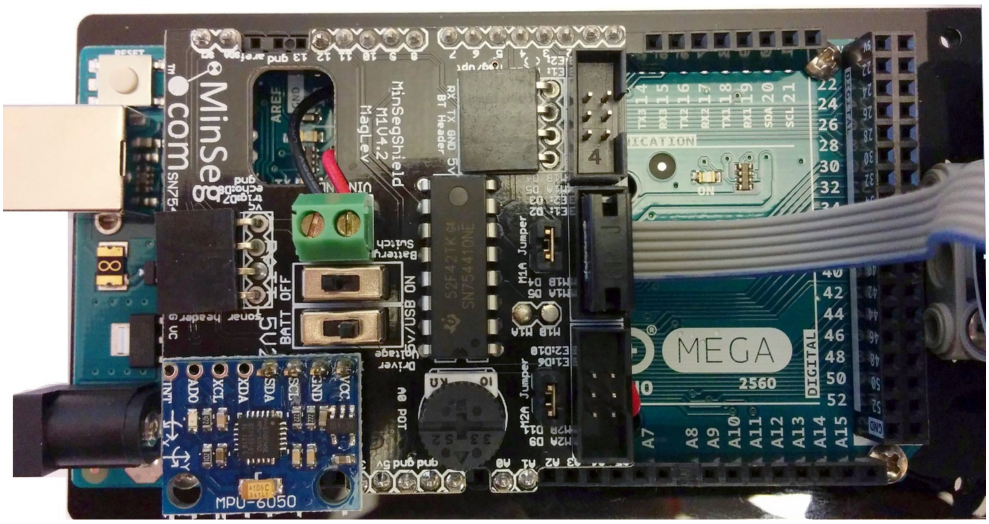

Figure 3.1: The accelerometer and gyro are placed on the blue board (MPU 6050) on the bottom left of the figure. The sensors are (with the setup in this lab) capable of measuring up to $4g$ accleration and an angular velocity of $250°/s$. The blue board combining an accelerometer and a gyro is called an IMU (inertial measurement unit). Retail price in the order of 50SEK. An IMU with similar performance was at least 100x as expensive 20 years ago, and 10 times as large.

applications, we are interested in the absolute angle of the device, such as knowing if the MinSeg is standing straight up or not. In addition to this, the sensor has bias errors, meaning that the angle estimate will drift. Even if we leave the device completely still, the preparations showed that any bias in the angular velocity measurement will lead to a growing error in the angle estimate.

In the lab, we are only studying the tilt-angle of the MinSeg, i.e., the balancing angle of the MinSeg relative to the surface ($0°$ degree when lying flat on a table, and $-90°$ when standing straight up). Consequently, we are only interested in rotations around the x-axis (rotations in the $y-z$ plane) and thus only use one of the gyro measurements.

The accelerometer

The accelerometer might seem unrelated to the orientation of the device, as it measures the three linear accelerations of the object. However,

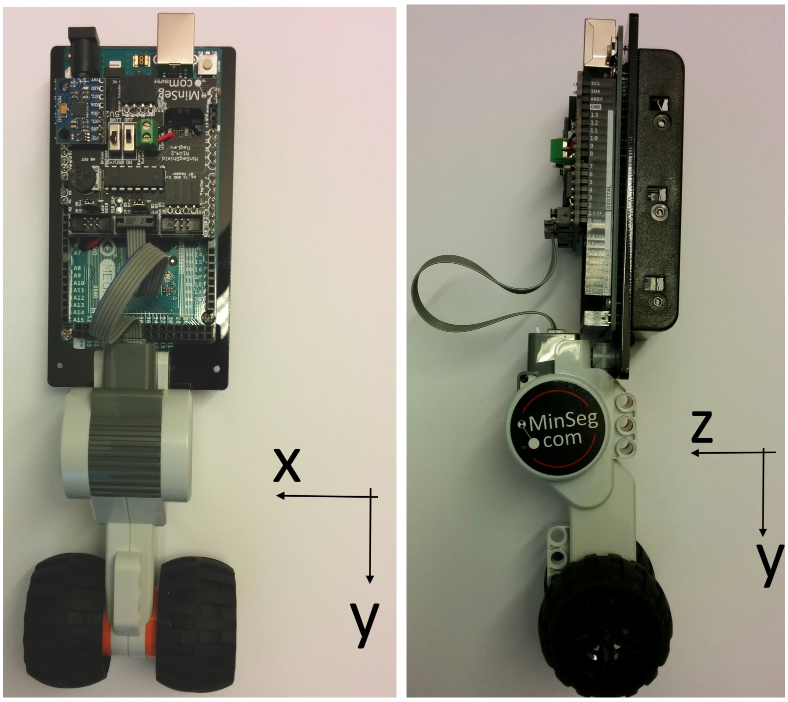

Figure 3.2: Body-fixed coordinate system used for accelerometer and gyro.

the acceleration is measured relative to free-fall, meaning that when the device is kept still, there should be an acceleration of magnitude $\sqrt{a_x^2 + a_y^2 + a_z^2} = 9.8 m/s^2$. The way this magnitude is distributed on the three body-fixed accelerations $a_x(t)$, $a_y(t)$ and $a_z(t)$ will give us information about the absolute orientation.

Consider the coordinate system of the setup in Figure 3.2. When the device is lying flat on a table with the battery holder facing down, we should see the measurements $a_x = 0$, $a_y = 0$ and $a_z = 9.8$ (The device is accelerating up from the table, relative to free-fall. An alternative view is that there is a force in the positive z-direction). When in perfect balancing mode, i.e., tilted up and standing on its wheels, the measurements should be $a_x = 0, a_y = -9.8$ and $a_z = 0$. The device is still accelerating up from the table relative to free-fall (normal force up from table), but y is pointing downwards, hence the negative sign.

From this, it is clear that when we are balancing the device on its wheels and it is sufficiently still, the tilt-angle is related to the relative size of the acceleration components a_y and a_z. No movement should occur in the x-directions, and thus a_x will be 0.

In practice the MinSeg is never completely still when balancing, so a_y and a_z will contain parts that come from actual movements, which will influence the computed angle, i.e., we will see noise on the angle estimate. Another problem is that the accelerometer might have been installed slightly tilted, which will give us a bias on the acceleration signals. However, since we never integrate the acceleration signals in this application, this bias will not lead to any kind of drift, but just a constant error on the angle estimate.

3.1.1 Gyro experiments

The first part of the lab will concentrate on the gyro, and in particular the gyro signal for rotation around the x-axis in the $y - z$ plane. Start MATLAB and the SIMULINK file `lab2template1`, shown in Figure 3.3.

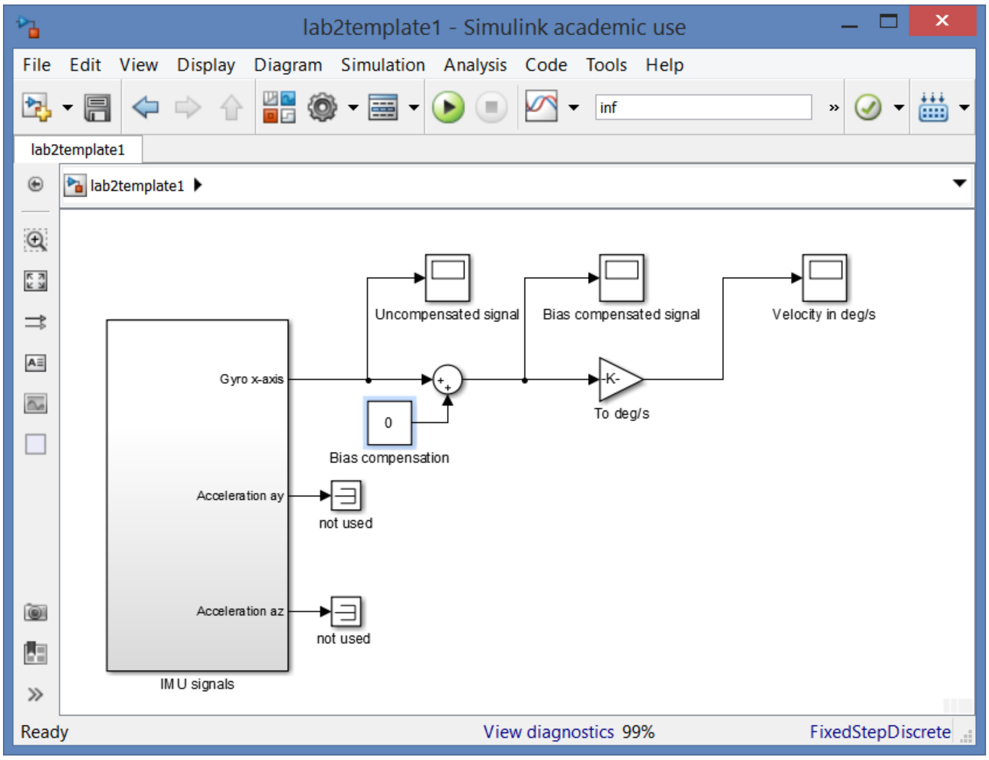

Figure 3.3: Template for gyro angle estimator.

Preparation 7 *The largest possible angular velocity the gyro can measure is* $250°/s$. *When the gyro senses this rate, it outputs the value* 32768 *(*2^{15}*, the largest possible number that can be encoded using* 16 *bits,* 1 *bit* *is used for the sign). How should the measurement be scaled to go from the measured integer value, to the unit deg/s.*

Task 1 *Check the value in the gain block to confirm your preparation on the measurement scaling.*

Task 2 *Assemble the MinSeg as shown on the front-page, and place the MinSeg flat on the table with the battery holder facing down, and connect the USB cable. Compile, download and run the code by pressing the green run button. Study the plot with the uncompensated measurement. As the MinSeg is stationary, the true angular velocity is* 0*, so everything you see is measurement errors (and perhaps extremely small rotations due to vibrations in the table). Make a rough estimate of the bias level, i.e., the constant error* b *in (2.1). Stop the code by pressing the square black box next to the run button.*

Preparation 8 *A simple way to reduce the bias effect is to estimate its value, and then alter the measurement so that the signal varies around* 0 *instead. Of course, this only works if the bias really is constant (it never is in practice, it might change with temperature etc), but it is a good start. If you see the gyro signal varying around the level* 70 *when lying still, what value should be used in the bias compensation block in Figure 3.3?*

Task 3 *Update the constant in the* `Bias compensation` *block such that the bias level is compensated for. Download and confirm the change by* *looking at the plot of the compensated signal. Also study the plot showing the angular velocity in degrees per second. How large is the amplitude of the fast varying measurement errors?*

We are now ready to start integrating the adjusted angular velocity measurements. To obtain the angle, all we have to do is to integrate the signal using the approximation in preparation exercise 3.

Task 4 *Extend your model according to Figure 3.4. The* ***Discrete-time Integrator*** *block can be found under* ***Discrete*** *in the library browser. Double-click the discrete-time integrator block in your model and change the Integrator method to Integration: Backward Euler, and change the sample-time to* -1 *(which means it uses the sample-time specified in the global model configuration). The Gain should be left at* 1.0*. Apply and close. You have now defined the integral to be computed exactly as in the preparation exercise.*

Task 5 *Start the code, and study the angle estimate plot. Note that it does not stay constant, despite the MinSeg being completely still. This is due to a remaining bias error and what you are seeing is mainly the linear drift. Find out how long it takes for the angle to drift* $1°$*. This is a typical performance measure on a gyro implementation. The longer it takes, the better. Your solution will probably be in the order of* $5-30$ *seconds, which means that after some minutes, the angle estimate is completely wrong. Rotate the MinSeg so that it stands on its wheels. The angle estimate should change* $-90°$*.*

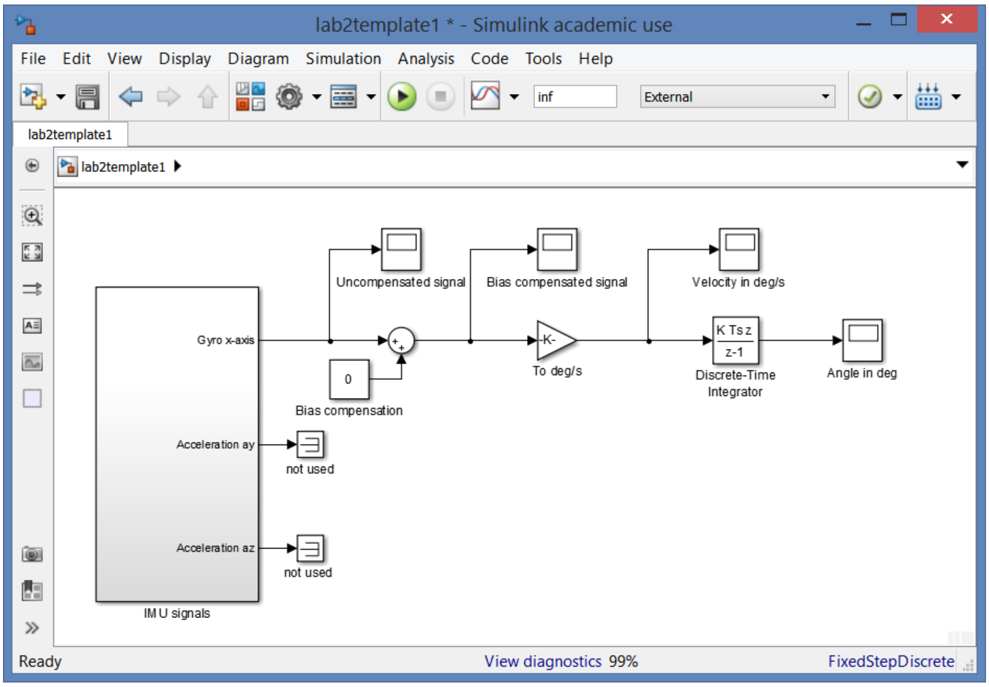

Figure 3.4: Integrated and bias-compensated gyro measurements.

3.1.2 Accelerometer experiments

Let us now turn to angle estimation from the accelerometer signals. Study Figure 3.5. If the device is stationary at an angle, the total acceleration is $g = 9.8m/s^2$ and it is pointing downwards, and thus the accelerometer measures g upwards (as it measures relative to free-fall). This acceleration signal will be picked up in the body-fixed coordinate system through the acceleration measurements $a_y(t)$ and $a_z(t)$. By geometry, we have $\theta(t) = \arctan(a_y(t)/a_z(t))$. Note that a_y will be negative from the definition of the coordinate system in Figure 3.2, hence the angle will be negative in the current definition.

Of course, if the MinSeg ever was in the angle illustrated in Figure 3.5, it would not be stationary but had to be moving. This means there might be additional forces and resulting accelerations acting, but the idea here is to see those effects as noise.

Task 6 *Open the file **lab2template2**. Double-click the gyro angle estimator and update the bias compensation to the value you are using. Download the code and study the angle estimate from the accelerometer*

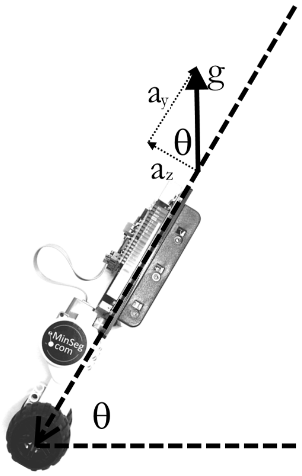

Figure 3.5: The acceleration relative to free-fall will be picked up in a stationary situation on the two coordinates $a_y(t)$ and $a_z(t)$ allowing us to compute the tilt-angle θ

signals, and compare it to the angle estimates from the gyro, which is displayed in the same plot. How do they compare and differ? See what happens if you move the MinSeg very quickly and go beyond the measurement limit $250°/s$ *in the gyro.*

3.1.3 Sensor fusion with complementary filter

What you should have seen now is that the angle estimated using the gyro behaves well on a short time-frame, but drifts on a long time-frame (low-frequency errors). The angle estimate from accelerometer signal on the other hand has no drift (good in a long time-frame), but has more noise (bad on a short time-frame, high-frequency errors). We would like to combine the merits from the two sensors, while avoiding the drawbacks of them. In other words, we trust the long-term average of the accelerometer angle estimate but not any of its fast changes, while we have no confidence in the long-term value of the gyro estimate.

Recall the estimate generated by the gyro signal. Call this estimate θ_g and consider the discrete-time computation with approximated integral.

$$\theta_g(kT_s) = \theta_g((k-1)T_s) + T_s\omega(kT_s) \tag{3.1}$$

If the estimate ever goes bad (due to drift for instance), it will never get better. If all movements end, the angle estimate will stay at its last value. What we could do is to bring it back to a somewhat correct value periodically. For instance, we could reset it to the value obtained from the accelerometer estimator every 10 seconds, or similar heuristics. A more clever idea is to continuously use the angle estimate from the accelerometers. Let the accelerometer angle estimate be called $\theta_a(kT_s)$, and introduce a new estimate $\theta_c(kT_s)$ which is a slight variation of the gyro integration

$$\theta_c(kT_s) = (1-\alpha)\theta_a(kT_s) + \alpha\big(\theta_c((k-1)T_s) + T_s\omega(kT_s)\big) \tag{3.2}$$

The constant α is typically close to 1. When it is 1, we obtain the gyro solution, and when it is 0 we obtain the accelerometer solution. What we are doing is that we are slowly pushing the estimate $\theta_c(kT_s)$ towards $\theta_a(kT_s)$, while still using the high-precision gyro measurements to detect rotations. Effectively, we will eliminate the drift, while retaining the good short time-frame qualities of the gyro. Using multiple sensors with different characteristics like this is called sensor fusion.

Preparation 9 *With z^{-1} denoting unit time-delay, show that (3.2) can be written as*

$$\boldsymbol{\theta}_c(z) = \frac{(1-\alpha)z}{z-\alpha}\boldsymbol{\theta}_a(z) + \frac{T_s\alpha z}{z-\alpha}\boldsymbol{\omega}(z) \tag{3.3}$$

Preparation 10 *With $\boldsymbol{\theta}_g(z) = \frac{T_s z}{z-1}\boldsymbol{\omega}(z)$ denoting our original gyro estimate, show that the solution (3.3) alternatively can be written as*

$$\boldsymbol{\theta}_c(z) = \frac{(1-\alpha)z}{z-\alpha}\boldsymbol{\theta}_a(z) + \frac{\alpha(z-1)}{z-\alpha}\boldsymbol{\theta}_g(z) \tag{3.4}$$

Hence, the new solution is a sum of the two old solution, each of them filtered. Let us look at the frequency response of those filters (here with $\alpha = 0.9$ and $T_s = 0.025s$)

```
  z = tf('z',0.025);
Ha = (1-0.9)*z/(z-.9);
Hg = 0.9*(z-1)/((z-.9));
bodemag(Ha,Hg);legend('Ha','Hg');
```

The result is seen in Figure 3.6. We see that the filter $H_a(z)$ used on the accelerometer angle estimate is a low-pass filter (removes fast variations and only keeps slow variations) and the filter $H_g(z)$ for the gyro angle estimate is a high-pass filter (removes the slow drift and only keeps fast variations). The name complementary filter comes from the fact that the two filters sum to 1 (check in (3.4)!).

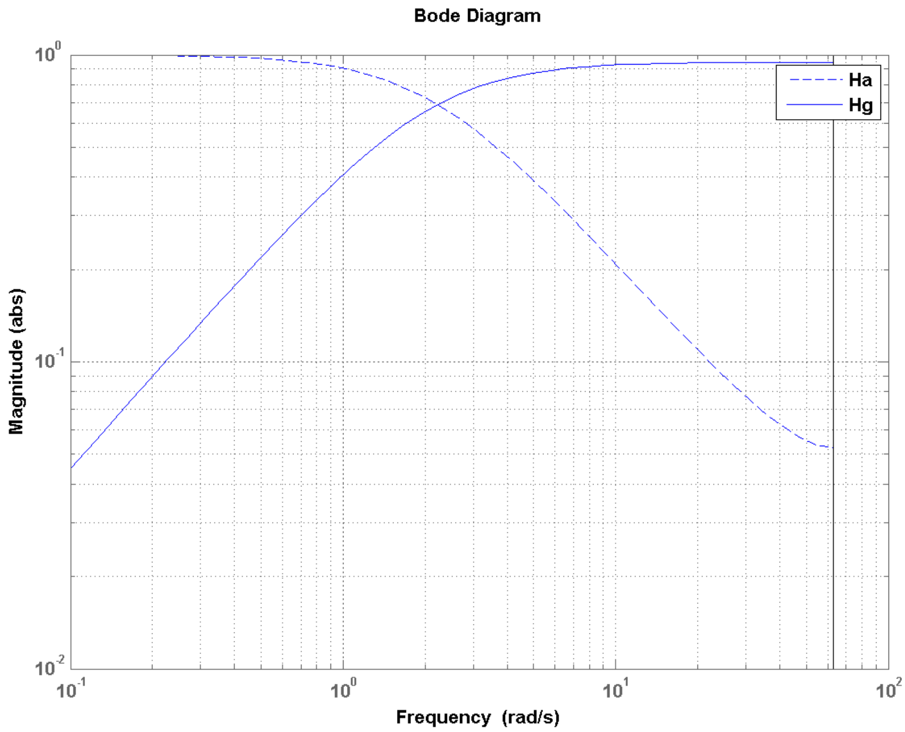

Figure 3.6: Amplitude gain for the two filters used in the complementary filter approach. The accelerometer solution is low-pass filtered, while the gyro solution is high-pass filtered

Preparation 11 *In the block diagram in Figure 3.7, values and connections are missing. Insert the values α, $1-\alpha$ and T_s in suitable blocks,* *and draw the connection, to implement the computation in (3.2). As a hint, standard Euler backwards as in (3.1) is implemented in Figure 3.8.*

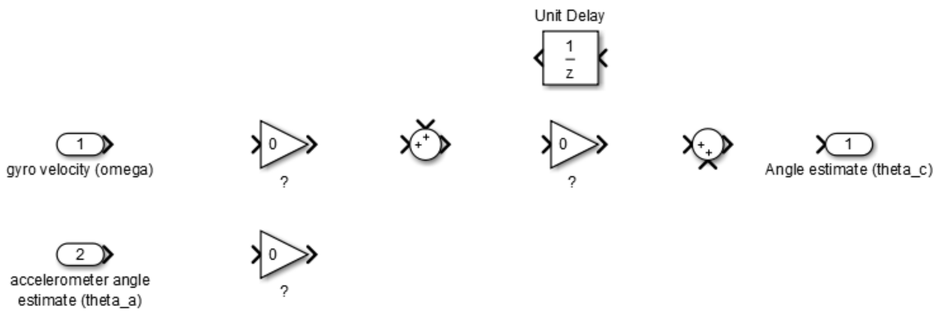

Figure 3.7: Insert missing values and connection to implement the complementary filter (3.3)

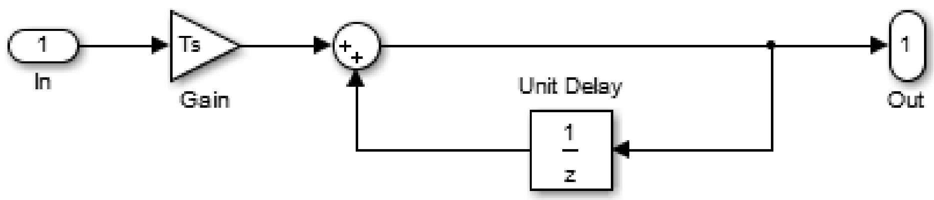

Figure 3.8: Example showing how discrete-time integral approximation (Euler backwards) would be implemented manually, $Out(kT_s) = Out((k-1)T_s) + T_s In(k)$

Task 7 *Open* ***lab2template3*** *and double-click the* ***Complementary filter*** *block, and complete it according to the preparation exercise. Use* $\alpha = 0.9$ *and* $Ts = 0.035$. *Update the bias compensation in the gyro angle estimator. Start the code and study the three angle estimate solutions. How do they compare? Try other values on* α *such as* 0.5*and* 0.99. *Try with a completely wrong value on the bias compensation (add an extra 500 or so). Rotate the MinSeg quickly so the* $250°/s$ *limit is violated.*

The angle estimate from the accelerometer signals are based on the raw accelerometer signals. An acceleration of 1 g corresponds to a measurement of 8192 while no acceleration should lead to the measurement 0. In our computation of the angle based on these signals, we have not scaled the data to m/s^2, as the scaling cancels out when we perform the division before taking arctan. However, we should compensate for calibration error / bias in the accelerometer signals.

Task 8 *In* ***Accelerometer angle estimator****, add bias compensation to the two measurement signals* $a_y(t)$ *and* $a_z(t)$ *to improve the accelerometer angle estimator. When lying flat on the table,* $a_y(t)$ *should be* 0*, and* $a_z(t)$ *should be* 8192*. You thus have to rebuild the model in the* ***Accelerometer angle estimator*** *block to incorporate bias compensation as in the* ***Gyro angle estimator*** *block. You do this exactly as when you tuned the gyro bias compensation. Can you improve the precision of the accelerometer angle estimator, and thus the complementary filter estimate?*