Introduction, The PID Controller

Automatic Control, Basic Course, Lecture 1

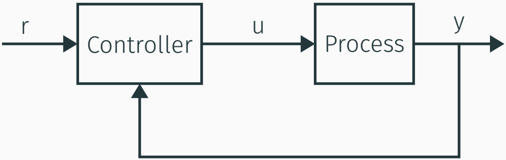
October 29, 2019

Lund University, Department of Automatic Control

Content

Introduction

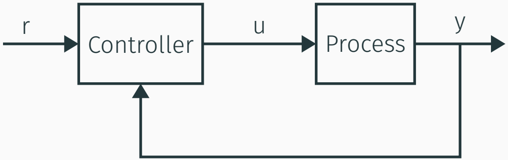
The Simple Feedback Loop



- Reference value **r**
- Control signal **u**
- Measured signal/output **y**

The problem/purpose: Design a controller such that the output follows the reference signal as good as possible

The Simple Feedback Loop



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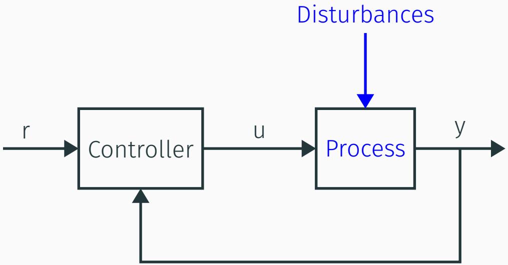
Note on terminology: Process, Controlled system, Plant etc...

The Feedback Loop



- Reference value **r**
- Control signal **u**
- Measured signal/output **y**

The problem/purpose: Design a controller such that the output follows the reference signal as good as possible despite disturbances and uncertainties in process.

Find the Control Problem - 1

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- Reference value - Desired temperature
- Control signal - e.g., power to the AC, amount of hot water to the radiators
- Measured value - The temperature in the room

Find the Control Problem - 2

Find the Control Problem - 2

- Reference value - Desired speed
- Control signal - Amount of gasoline to the engine
- Measured value - The speed of the car

Find the Control Problem - 3

Find the Control Problem - 3

- Reference value - Number of bacterias
- Control signal - “Food” (sugar and O_2)
- Measured value - E.g., pH or oxygen level in the tank

Feedforward

Feedforward: Analyze and determine on **beforehand** what to do and how to react.

Benefits/Drawbacks with feedforward:

+ Reduces effect of **measurable** disturbances
+ Allows for fast reference changes without introducing a control error
- Demands accurate model of the process
- Demands stable systems

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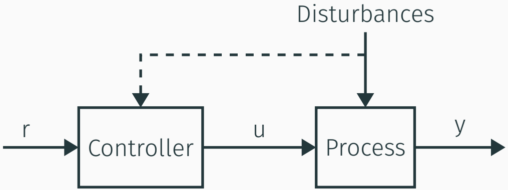
- Demands accurate model of the process
- Demands stable systems

The term **feedforward** can be either the shape of the reference signal determined a priori (e.g., look-up table or a predetermined way to compensate for measurable disturbances.

Feedforward

Some systems can operate well without feedback, i.e., in open loop.



Examples of open loop systems?

Feedforward vs. Feedback

Benefits/Drawbacks with feedback:

+ Stabilize unstable systems
+ The speed of the system can be increased
+ Less accurate model of the process is needed
+ Disturbances can be compensated
- **WARNING:** Stable systems might become unstable with feedback

Feedforward vs. Feedback

Benefits/Drawbacks with feedback:

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- \- **WARNING:** Stable systems might become unstable with feedback

Feedforward and feedback are **complementary** approaches, and a good controller typically **uses both**.

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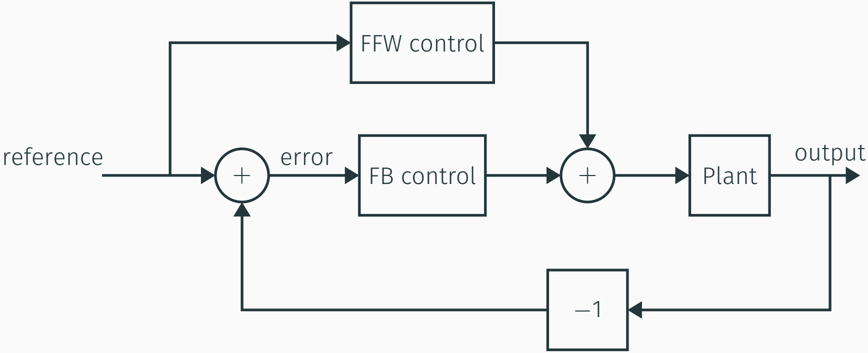

The Magic of Feedback

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[Clarke's 3rd law]

—"Any sufficiently advanced technology is indistinguishable from magic".

Arthur C. Clarke, "Profiles of The Future", 1961

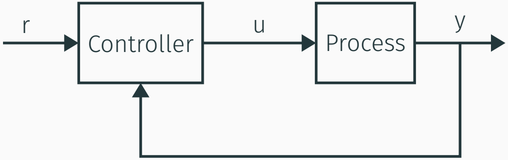
English physicist & science fiction author

The PID Controller

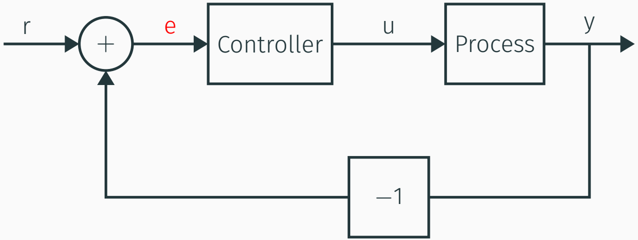
The Error

The input to the controller will be the **error**, i.e.,
the difference between the reference value and the measured value.

$$e = r - y$$

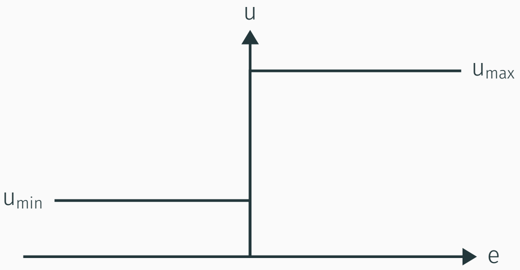


New block scheme:



On/Off Controller

$$u = \begin{cases} u_{max} & \text{if } e > 0 \\ u_{min} & \text{if } e < 0 \end{cases}$$



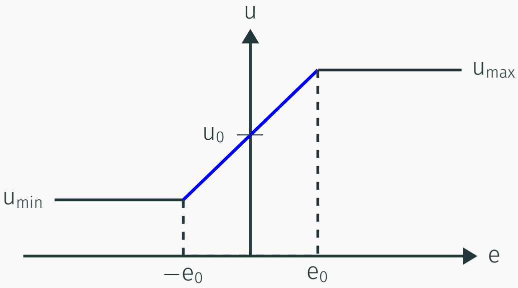
Usually not a good controller. Why?

The P-part

Idea: Decrease the controller gain for small control errors.

P-controller:

$$u = \begin{cases} u_{max} & \text{if } e > e_0 \\ u_0 + Ke & \text{if } -e_0 \le e \le e_0 \\ u_{min} & \text{if } e < -e_0 \end{cases}$$



P-part comes from proportional (here affine) to the error e.

The P-part

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The control error

$$e = \frac{u - u_0}{K}$$

To have $e = 0$ at stationarity, either:

- $u_0 = u$
- $K = \infty$

The P-part

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The control error

$$e = \frac{u - u_0}{K}$$

To have $e = 0$ at stationarity, either:

- $u_0 = u$ (What if u varies?)
- $K = \infty$ (On/off control)

The I-part

Idea: Adjust u_0 automatically to become u.

PI-controller:

$$u(t) = K\left(\frac{1}{T_i}\int^t e(\tau)\mathrm{d}\tau + e\right)$$

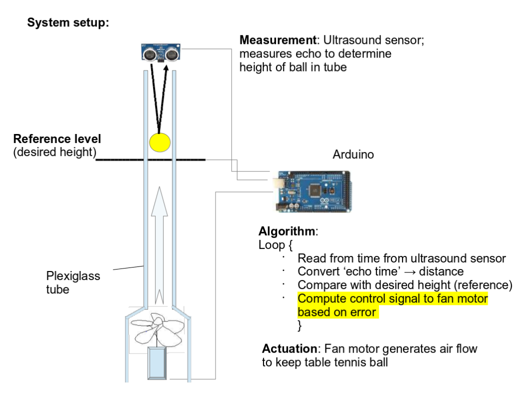
Compared to the P-controller, now

$$u = u_0 + Ke, \qquad u_0(t) = \frac{K}{T_i}\int^t e(\tau)\mathrm{d}\tau$$

At stationary $e = 0$ if and only if $r = y$.

PI controller achieves what we want, if performance requirements are not extensive.

Example of integral action needed — mini-problem (5 min)



(a) Argue why there will be a stationary error if we just use P-control; i.e., $u(t) = K \cdot (h_{ref} - h)$?

(b) How will the stationary error change with the value of the gain K?

(c) What happens if we add integral action with very small integral gain $\frac{K}{T_i}$? Sketch the behaviour.

Answer mini-problem

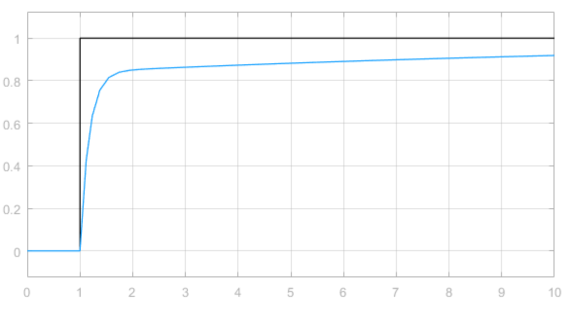
Note: This is not a strict answer and you need to make reasonable assumptions about the process yourself for this to hold.

(a) Argue why there will be a stationary value if we just use P-control; i.e., $u(t) = K \cdot (h_{ref} - h)$?
If $h = h_{ref}$ the control signal $u(t) = K \cdot (h_{ref} - h) = 0$ and the motor shuts off/fan stops spinning and the ball will fall. The process will finally settle to an equilibrium with a positive stationary error $e = h_{ref} - h$ such that the corresponding control signal will keep the ball at a fixed error (e) from the reference.

(b) How will the stationary value change with the value of the gain K?
The control signal to the fan motor $u = K \cdot e$ is the product of the gain and the error; for a higher gain K you can reach stationarity with a smaller stationary error e.

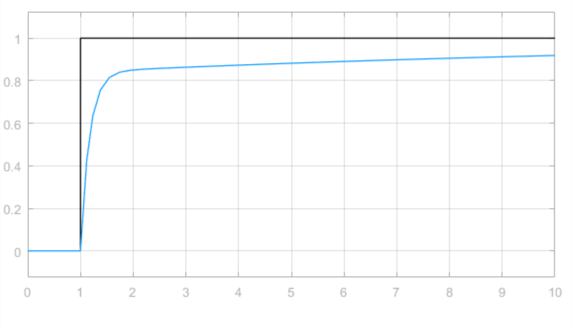
Answer mini-problem, cont'd

(c) What happens if we add integral action with **very small integral gain** $\frac{K}{T_i}$? Sketch the behaviour.



Answer mini-problem, cont'd

(c) What happens if we add integral action with very small integral gain $\frac{K}{T_i}$? Sketch the behaviour.



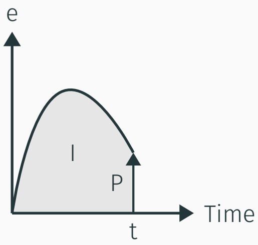
Note how the height of the ball (slowly) approaches the desired reference (as the integral part makes the control action increase as long as there is an error).
See also separate simulink example/demo.

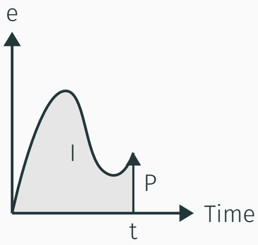
The D-part

Idea: Speed up the PI-controller by “looking ahead”/”predicting future”.

PID-controller:

$$u = K\left(e + \frac{1}{T_i}\int^t e(\tau)\mathrm{d}\tau + T_d \frac{\mathrm{d}e}{\mathrm{d}t}\right)$$





Same P- and I-part in both cases, but **very different behavior** of error. The derivative of e contains a lot of information to utilize.

- P acts on the current error,
- I acts on the past error,

The D-part

Idea: Speed up the PI-controller by “looking ahead”/”predicting future”.

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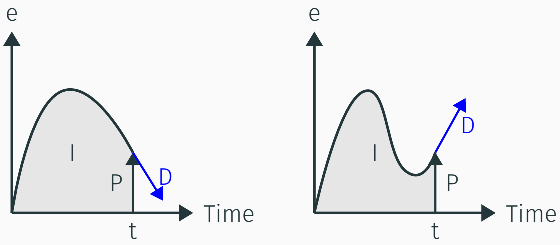

Same P- and I-part in both cases, but **very different behavior** of error. The derivative of e contains a lot of information to utilize.

- P acts on the current error,
- I acts on the past error,
- **D acts on the ”future”/predicted error.**

Next Lecture: Process Models. Linearization