LUNDS TEKNISKA HÖGSKOLA
Lunds universitet

Institutionen för
REGLERTEKNIK

System Identification

Final Exam March 11, 2005, 8-13

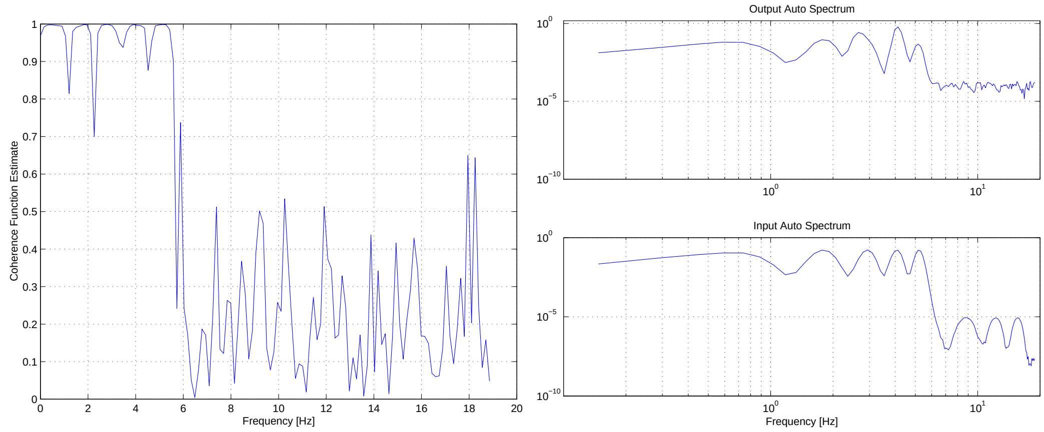
General Instructions

This is an open book exam. You are allowed to use any book you want. No notes or solution manuals are allowed. Solutions and answers to the problems should be well motivated. The exam consists of 7 problems. The credit for each problem is indicated in the problem. The total number of credits is 25 points. Preliminary grade limits:

Grade 3:	12 – 16 points
Grade 4:	17 – 21 points
Grade 5:	22 – 25 points

Results

The results of the exam will be posted at the latest March 18 on the note board on the first floor of the M-building and they will also be available on the course home page. The exams are, after this date, on display in Brad Schofield's office in the M-building, Level 2, on March 18, 12.00 – 13.00.



Figur 1 Left: coherence function $\gamma(\omega)$. Right: input and output autospectra S_{uu} and S_{yy}.

1. Figure 1 shows the coherence function $\gamma(\omega)$ and the input and output autospectra S_{uu} and S_{yy} from an identification experiment conducted on the system given by:

$$Y(s) = G(s)U(s) + N(s)$$

where the input $U(s)$ and noise $N(s)$ are known to be uncorrelated. The aim of the identification is to design a controller for the process. The data sets $\{u_k\}$ and $\{y_k\}$ from the experiment are used to obtain an ARMAX model of the system. Pole placement design is used to obtain a controller giving a closed-loop bandwidth of 20π radians per second.

a. The coherence function $\gamma(\omega)$ between signals u and y is defined as

$$\gamma(\omega) = \frac{|S_{yu}(i\omega)|}{\sqrt{S_{uu}(i\omega)S_{yy}(i\omega)}}.$$

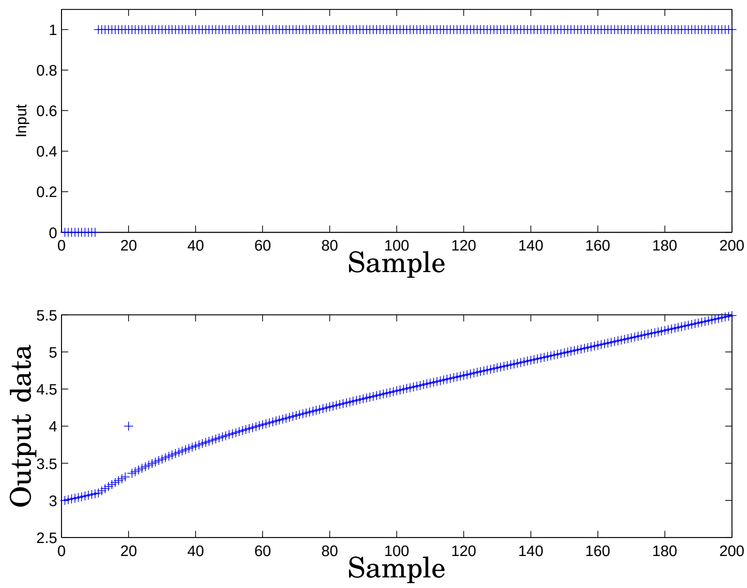
Express $\gamma(\omega)$ in terms of the noise autospectrum S_{nn} and the input autospectrum S_{uu} in such a way that the excitation of the input signal may be analysed. (2 p)

b. When the controller described above is tested, it is found to work very poorly. Explain why this might be the case. (2 p)

c. Suggest an improvement to the experimental procedure which may improve the performance of the controller. (1 p)

2. Consider a system with time-varying parameters. Identification of the model is performed using recursive least-squares (RLS) estimation with forgetting factor λ

$$\begin{aligned} \widehat{\theta}_k^{RLS} &= \widehat{\theta}_{k-1}^{RLS} + K_k(y_k - \varphi_k^T \widehat{\theta}_{k-1}^{RLS}) \\ K_k &= P_k \varphi_k \\ P_k &= \frac{1}{\lambda}\left(P_{k-1} - \frac{P_{k-1}\varphi_k\varphi_k^T P_{k-1}}{\lambda + \varphi_k^T P_{k-1}\varphi_k}\right) \end{aligned}$$



Figur 2 Output data from a experiment.

which asymptotically minimizes the loss function

$$V(\bar{\theta}, k) = \frac{1}{2} \sum_{i=1}^{k} \lambda^{k-i} (y_i - \varphi_i^T \bar{\theta})^2$$

What value should the forgetting factor have if data older than 100 samples should be weighted by less than 0.15 in the loss function? (1 p)

3. Consider the system:

$$\mathcal{S} : \quad y_k = \phi_k^T \theta + v_k$$

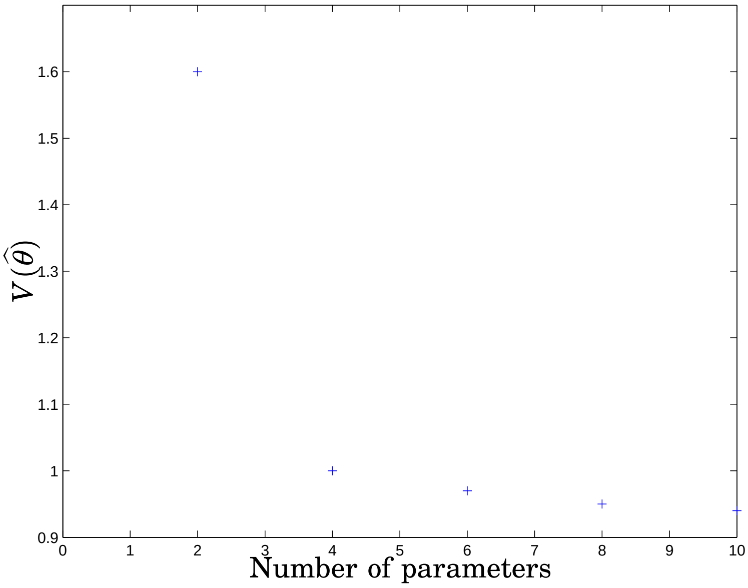
where v is a white noise sequence with zero mean and known variance σ^2. Each v_k has the probability density function:

$$f_e(v_k) = \frac{1}{\sqrt{2\pi}\sigma} e^{-v_k^2/2\sigma^2}$$

Determine the loss function minimized when the maximum likelihood (ML) method is used to estimate the parameter vector θ. How does this loss function compare to the loss functions of other identification methods? Draw conclusions. (3 p)

4 a. You have performed a step response experiment and obtained both input and output data as shown in Figure 2. Discuss if the sampling rate has been sufficiently high to capture the dynamics. (1 p)

b. What shall be done with the output data shown in Figure 2 before the identification procedure starts? (2 p)



Figur 3 The loss function.

c. Cross-validation is an important part of the model validation process. Discuss how the output data should be arranged for cross-validation and what amplitude of the input signal should be used. (1 p)

5. Consider an unknown linear system

$$\mathcal{S} \; : \qquad y_{k+n} + a_1 y_{k+n-1} + \cdots + a_n y_k = b_1 u_{k+n-1} + \cdots + b_n u_k + e_{k+n}$$

where $e_k \in \mathcal{N}(0,1)$. We have performed an experiment collecting 200 data points where the input u was a PRBS signal with unit amplitude. We want to estimate ARX models, that is,

$$\mathcal{M} \; : \qquad y_{k+n} + a_1 y_{k+n-1} + \cdots + a_n y_k = b_1 u_{k+n-1} + \cdots + b_n u_k$$

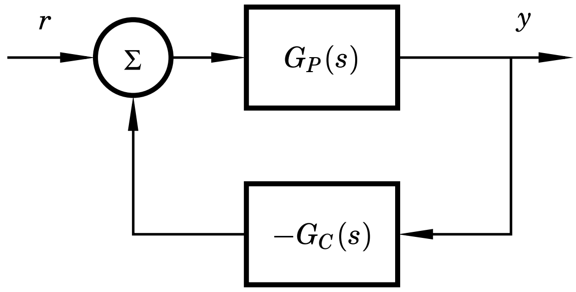
where $n = 1, \ldots, 5$.

Determine the suitable number of parameters the ARX-model should have, using the Akaike information criterion and the final prediction error criterion if the loss function is given by

$$V(\widehat{\theta}) = \frac{1}{N} \sum_{k=1}^{N} \frac{1}{2} \varepsilon_k^2(\widehat{\theta}), \qquad \varepsilon_k(\widehat{\theta}) = y_k - \varphi_k^T \widehat{\theta}$$

$$V(\widehat{\theta}) = [1.6 \quad 1.0 \quad 0.97 \quad 0.95 \quad 0.94] \quad \text{for} \quad n = 1, \ldots, 5$$

and is shown in Figure 3. (2 p)



Figur 4 The system in Problem 7.

6 a. What is meant by the term *balanced realization*? (1 p)

b. Show that:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -0.05698 & -0.1914 \\ -0.1914 & -0.643 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} -0.9998 \\ 0.01877 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} -0.9998 & 0.01877 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

is a state-space realization of the system:

$$H(z) = \frac{z + 0.65}{z^2 + 0.7z}$$

(1 p)

c. The realization above is in fact balanced. Use this information to determine the asymptotic reachability and observability Gramians P and Q. (2 p)

d. Use the Gramians to determine whether is is advisable to reduce the order of the model. If so, determine the reduced-order model. (2 p)

7. Closed-loop identification is sometimes necessary, for example in cases where the plant is unstable or when using operational data records from plants in production. Assume that we have performed indirect identification and obtained the following closed-loop model

$$G_{cl}(s) = \frac{(s+1)(s+3)}{(s+2)(s+3)(s+4)+1}$$

Figure 4 shows the configuration of system that we have identified. We know the controller which has been used in the experiment

$$G_C(s) = \frac{1}{s+3}$$

a. Find a model for the process $G_P(s)$ (2 p)

b. Discuss the main possible transfer function estimation problem associated with indirect identification. (1 p)

8. Assume that the following impulse response data have been obtained

$$\{0, 1, 0.5, 0.25, 0.125, 0.0625, ...\}$$

Use the Ho-Kalman algorithm (or Kung's algorithm) to find a state-space model reproducing the impulse response data. (1 p)