LUND INSTITUTE
OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

System Identification

Final Exam March 9, 2007, 8-13

General Instructions

This is an open book exam. You are allowed to use any book you want. No notes or solution manuals are allowed. Solutions and answers to the problems should be well motivated. The exam consists of 8 problems. The credit for each problem is indicated in the problem. The total number of credits is 25 points. Preliminary grade limits:

Grade 3:	12 – 16 points
Grade 4:	17 – 21 points
Grade 5:	22 – 25 points

Results

The results of the exam will be posted at the latest March 16 on the note board on the first floor of the M-building and they will also be available on the course home page. The exams are, after this date, on display in Maria Karlsson's office in the M-building, Level 2.

1.

a. You are hired to identify a process in a chemical plant. After some investigation, you can tell that the only dynamics in the process is a time delay τ, so that the discrete time model

$$y_k = bu_{k-1} + e_k$$

is appropriate, when the sampling time h is taken to be equal to the process delay. The disturbance entering the system is assumed to be white noise with mean $E(e_k) = \mu$ and variance $var(e_k) = \sigma_e^2$.

You are given a data set consisting of 1000 samples from the process. Use the least-squares-method to find estimates of b, μ and σ_e^2. Some information from the data set is given below.

$$\sum_{k=1}^{N-1} u_k = 210.4 \qquad \sum_{k=1}^{N-1} y_{k+1} = 1054.9$$

$$\sum_{k=1}^{N-1} u_k^2 = 51.0 \qquad \sum_{k=1}^{N-1} y_k^2 = 1277.9$$

$$\sum_{k=1}^{N-1} u_k u_{k+1} = 45.0 \qquad \sum_{k=1}^{N-1} y_k y_{k+1} = 1126.8$$

$$\sum_{k=1}^{N-1} u_k y_{k+1} = 230.6 \qquad \sum_{k=1}^{N-1} y_k u_{k+1} = 255.5$$

(3 p)

b. Under which conditions on the sequence $\{u_k\}$ is the estimation method used in (a) consistent? (1 p)

c. Which of the following choices of $\{u_k\}$ fulfill the condition you derived in (b)?

1. $u_k = 5$
2. $u_k = \sin(\pi k/2)$
3. $u_k = \begin{cases} 0, k \leq 10 \\ 10 \;\; k > 10 \end{cases}$
4. $u_k = 0.2y_{k-1}$

(2 p)

2. For identification in closed-loop, the choice of reference signal is important. Consider the following options for r_k:

1. constant
2. gaussian white noise sequence
3. a sequence of sinusoidal signals of different frequencies
4. PRBS signal

Without any knowledge of the process to be identified, which signal would you prefer? For each of the other options, explain why your choice is superior. (2 p)

3. A system is modelled by the transfer function

$$H(z) = \frac{z - 0.9}{(z - 0.5)(z - 0.91)} \tag{1}$$

A balanced realization of the system is given by

$$\begin{aligned} x_{k+1} &= \begin{pmatrix} 0.5208 & -0.090 \\ -0.090 & 0.8892 \end{pmatrix} x_k + \begin{pmatrix} -0.9975 \\ -0.070 \end{pmatrix} u_k \\ y_k &= \begin{pmatrix} -0.9975 & -0.070 \end{pmatrix} \end{aligned}$$

a. Find the observability Gramian of the balanced realization model. Use balanced model reduction to find a first-order model of the system. (2 p)

b. Find a first-order model by an appropriate pole-zero-cancellation in the transfer function model (1), with compensation for the static gain. Compare with your result in (a). (1 p)

4. In figure 1, data from an identification experiment for an HCCI engine is shown. The input u is the crank angle degree of closing the inlet valve, and the output y is the crank angle degree of combustion. For control design, we wish to find a state-space model of the form

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Kw_k \\ y_k &= Cx_k + w_k \end{aligned}$$

Explain why we cannot expect to find a good model of this form for this process.

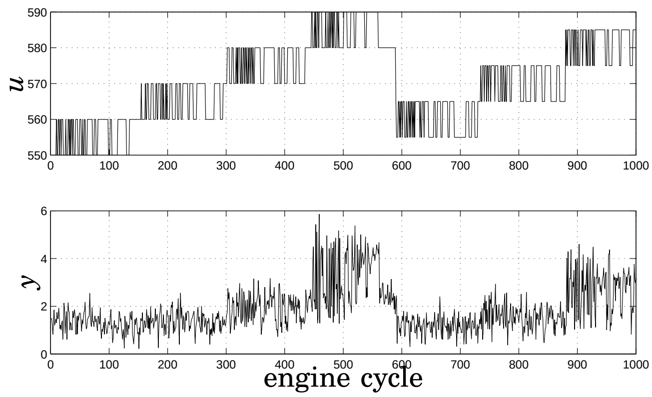

Figure 1

(1 p)

5.

a. An n-th order ARX-system is given by

$$y_k = \begin{pmatrix} -y_{k-1} & \cdots & -y_{k-n} & u_{k-1} & \cdots & u_{k-n} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_n \\ b_1 \\ \vdots \\ b_n \end{pmatrix} + v_k = \phi_k \theta + v_k \quad (2)$$

The noise $\{v_k\}$ is assumed to be exponentially distributed white noise with probability density function

$$f_v(v) = \frac{1}{\sqrt{2}\sigma} e^{-\sqrt{2}|v|/\sigma}$$

Show that the maximum-likelihood estimate of θ can be found by solving the optimization problem

$$\min_\theta V(\theta)$$

where the cost function $V(\theta)$ is given by

$$V(\theta) = \sum_{i=n+1}^{N} |y_k - \phi_k \theta| \quad (3)$$

(2 p)

b. Data of length $N = 3000$ was collected from a third-order process of the form (2).The optimization problem (3) was solved numerically, yielding the optimum

$$\hat{\theta}_{ML} = \begin{pmatrix} -0.830 & 0.972 & -0.243 & 0.52 & -0.59 & 0.175 \end{pmatrix}$$
$$V(\hat{\theta}_{ML}) = 1823$$

Find the maximum-likelihood estimate of σ, the standard deviation of the noise v. (2 p)

6. Input u and output y from an identification experiment is shown in figure 2, along with the coherence function γ_{uy}. The data is to be used to estimate a non-parametric model of the process in the form of a Bode diagram. The discrete Fourier transforms $U_N(i\omega)$ and $Y_N(i\omega)$ of u and y were computed. An estimate of the process transfer function $H(e^{i\omega h})$ was computed according to

$$\hat{H}(e^{i\omega h}) = \frac{Y_N(i\omega)}{U_N(i\omega)}$$

The Bode diagram of $\hat{H}(e^{i\omega h})$ is shown in figure 3.

This Bode diagram was used for control design with a closed-loop bandwidth of $\omega_c = 100$ rad/s, but with disappointing results.

Suggest at least two modifications to the experimental setup and/or the identification algorithm that may improve the estimation of the Bode diagram. (2 p)

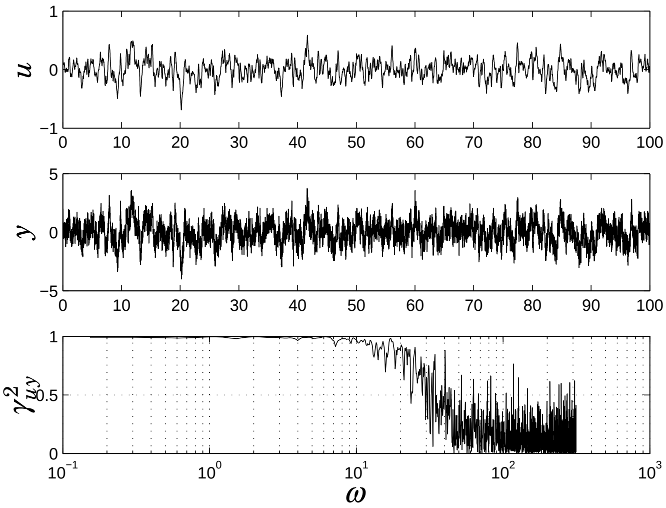

Figure 2

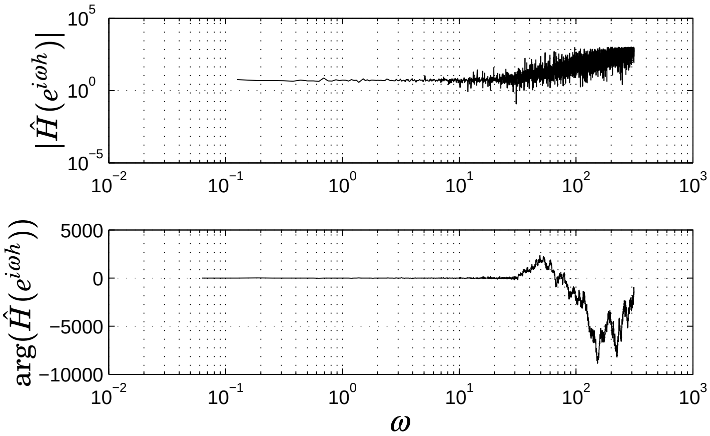

Figure 3

7. The Fibonacci numbers are given by the sequence

$$\{y_0, y_1, \ldots\} = \{1, 1, 2, 3, 5, 8, 13, 21, \ldots\}$$

Use the Ho-Kalman algorithm to find a state-space system that has the Fibonacci number sequence as its impulse response.

You may use the following result. The singular value decomposition of the matrix

$$X = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 8 \end{pmatrix},$$

$X = U\Sigma V^T$ is given by

$$U = \begin{pmatrix} -0.31 & 0.76 & -0.58 \\ -0.50 & -0.64 & -0.58 \\ -0.81 & 0.11 & 0.58 \end{pmatrix} \qquad \Sigma = \begin{pmatrix} 12.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.31 & -0.76 & -0.58 \\ -0.50 & 0.64 & -0.58 \\ -0.81 & 0.11 & 0.58 \end{pmatrix}$$

(3 p)

8.

a. An ARMA-system is given by

$$\mathcal{S} : y_k = -ay_{k-1} + e_k + ce_{k-i}$$

for some value of $i > 0$, where $\{e_k\}$ is white Gaussian noise with $E(e_k) = 0$, $var(e_k) = \sigma_e^2$. The parameter a is estimated from data from $\mathcal{S}$. Assume the parameter estimate is correct, i.e. $\hat{a} = a$, and that $|a| < 1$. Compute the autocovariance function $C_{\epsilon\epsilon}(\tau)$ of the residual sequence $\epsilon_k = y_k - \phi_k\theta$.

(2 p)

b. Assume $i = 1$. Data of length $N = 5000$ was collected from the system $\mathcal{S}$. The residual sequence

$$\epsilon_k = y_k + ay_{k-1}$$

was computed. Consider the hypotheses

$$\mathcal{H}_0 : c = 0$$
$$\mathcal{H}_1 : c \neq 0$$

The number of zero-crossings for the residual sequence in the data set was $\tau = 2642$. Based on the number of zero-crossings, is it possible to reject $\mathcal{H}_0$ on the significance level $p = 0.01$? (2 p)